Redundancy Resolution Schemes for Kinematically Redundant Parallel Manipulators

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Abstract. Kinematic redundancy may be an alternative for enlarging the workspace and for improving the dynamic performance of parallel manipulators. It can be implemented by the introduction of an extra active joint in an active kinematic chain. Kinematically redundant parallel manipulators requires redundancy resolution schemes since their inverse kinematic problem presents infinite solutions. These schemes can be posed as an optimization problem that can be solved locally or globally. These work compares numerical and experimental results of local and global redundancy resolution schemes applied to a three-level kinematically redundant 3$\underline{\text{PR}}$RR parallel manipulator. The results demonstrate that for the execution of predefined task, global strategies present improved results.

Keywords: parallel kinematic machines, redundancy resolution schemes, kinematic redundancy

1 Introduction

Parallel kinematic machines (PKMs) are composed of lighter mechanical components due to their kinematic architecture. Thus, this choice can be a promising alternative for designing manipulators with improved dynamic performance [1, 2] and more energetically efficient [3]. Nevertheless, PKMs suffer from important drawbacks such as the presence of singularities [4]. Singular regions in the workspace can be reduced by the inclusion of kinematic redundancies. They can be implemented by the introduction of extra active joints in a kinematic chain [4].

The inverse kinematic problem of kinematically redundant manipulators presents infinite solutions. Schemes for performing a suitable selection among these possibilities are denoted as redundancy resolution strategies [5]. These strategies can be formulated as an optimization problem that can be solved locally or globally [6]. The local strategies address the problem by considering the kinematic relations which can be formulated using, for instance gradient projection and Jacobian-based methods [5]. Boudreau & Nokleby [7] have proposed local optimization problems exploiting the kinetostatic analysis to find the inputs of the redundant actuators of kinematically redundant planar parallel manipulators. Whislt, the global strategies, also denoted as tracking problems [6], aim to find the optimal inputs for a predefined task. These methods have been exploited by Kotlarski *et al.* [8] for dealing with a single level of kinematic redundancy in a planar PKM. Extra levels of redundancies have been numerically and experimentally evaluated by Fontes *et al.* [2, 9, 3] also using global strategies for minimizing the maximum required torque and maximizing the distance to singular regions. Redundancy resolution schemes for parallel manipulators with several levels of kinematic redundancy yield cumbersome optimization problem due to the number of optimization variables and the complexity of the kinematic relations. In order to reduce this complexity, the task is usually described by a polynomial function [2, 8, 9]. This description decreases the versatility of the redundancy resolution schemes.

In this work, two redundancy resolution schemes are exploited for deriving the inputs of the redundant actuators of a 3PRRR planar parallel manipulator for a tracking trajectory problem. The first scheme is a direct global one that attempts to minimize a weighted cost function. The inputs of the redundant actuators are mathematically described by a polynomial function [8, 9]. The second scheme is a two-stage global one. The first stage is based on a gradient projection method. The outcome of this stage may demand unrealistic input values. To overcome this issue, a second stage has been proposed by Santos & da Silva [10]. This second stage uses Differential Dynamic Programming [11] to minimize the difference between the outcome of this first stage from an optimal solution considering limited accelerations. Figure 1 depicts the 3PRRR which is composed of three kinematic chains. Each one has an active prismatic joint (P), an active revolute joint (R) and two passive revolute joints (RR).

This manuscript is organized as follows. The prototype and its models are described in Section 2. Section 3 describes the metrics exploited for the evaluation of the posture of the manipulator's end-effector. Based on these metrics, the redundant resolution schemes are described in Section 4. Experimental results are discussed in Section 5 and conclusions are drawn in Section 6.

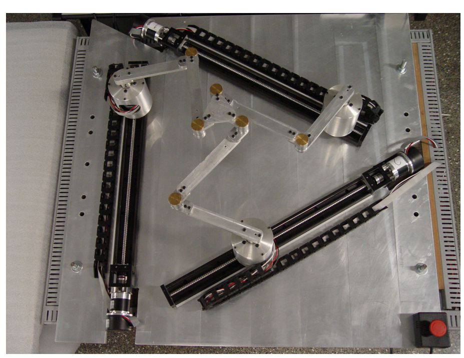

Fig. 1. 3PRRR: the kinematically redundant planar parallel manipulator

2 $3\underline{\text{PR}}\text{RR}$: prototype and modeling

Three redundancy resolutions schemes are exploited in this manuscript for deriving the inputs of the redundant actuators of a $3\underline{\text{PR}}\text{RR}$ manipulator. The metrics for evaluating this robotic system requires the calculation of its kinematic and dynamic models. The prototype experimental setup and its models are briefly introduced in this section. A complete derivation of these models can be found in [9, 10].

2.1 Setup Description

The $3\underline{\text{PR}}\text{RR}$ is actuated by six servomotors, depicted in Fig. 1, composed by a brushless Maxon EC60 flat motor, a Maxon planetary gearhead GP52C and a controller board Maxon EPOS2 50/5 yielding 0.82N.m @ 1200rpm of nominal torque. The linear motion is performed by three table systems (HIWIN KK60-10-C-E-600-A-1-F0-S3). Their stroke range is 600mm and their pitch is 10mm. The communication between the boards is via CAN protocol and the communication with the computer is via USB port. For the experimental campaign, the user provides the desired positions and velocities at diverse time instants. This input is interpolated and used as a reference signal to a linear position feedback control strategies. The gains have been adjusted manually.

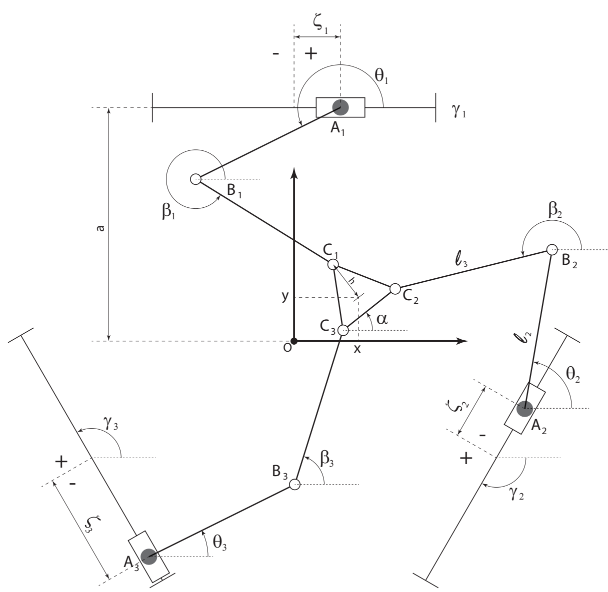

Fig. 2. Geometric details of the $3\underline{\text{PR}}\text{RR}$ [10]

2.2 Kinematics

The Inverse Kinematic Model (IKM) is the set of kinematic relations that relates the active joints' inputs $\boldsymbol{\Theta}$ for a given end-effector's position $\mathbf{X} = [x, y, \alpha]^T$. The active joints' inputs for the non-redundant manipulator can be described the following vector $\boldsymbol{\Theta} = [\theta_1, \theta_2, \theta_3]$, while for the 3$\underline{\text{PR}}$RR this vector is described by $\boldsymbol{\Theta} = [\theta_1, \theta_2, \theta_3, \zeta_1, \zeta_2, \zeta_3]^T$.

Figure 2 shows a scheme of the geometry of the manipulator under investigation. In this figure, the inputs of the active revolute joints are represented by θ_i and the inputs of the active prismatic joints by ζ_i where i represents each kinematic chain ($i = 1 \dots 3$). The end-effector's posture can be described by x, y and α. A_i is the active revolute joint, whilst B_i and C_i are passive ones. The lengths of links A_iB_i and B_iC_i are, respectively, l_2 and l_3. The angles γ_i and β_i represent the orientation of each prismatic joint and the links B_iC_i. The length h is the distance between the centroid of the end-effector and the point C_i. A coordinate system $O - (x, y)$ is defined at the center of the manipulator. The distance between the origin of this coordinate system and the center of any prismatic actuator is a.

The IKM can be derived by the following geometrical constraint [10]:

$$||\mathbf{r}_{\mathbf{C_i}} - \mathbf{r}_{\mathbf{B_i}}||^2 = \left\| \begin{bmatrix} \rho_{xi} - l_2 \cos\theta_i \\ \rho_{yi} - l_2 \sin\theta_i \end{bmatrix} \right\|^2 = l_3^2, \tag{1}$$

where $\mathbf{r}_{\mathbf{B_i}}$ and $\mathbf{r}_{\mathbf{C_i}}$ are the position vectors, $\rho_{xi} = x + h(\cos(\alpha + \gamma_i \pm {}^{\pi}/_2) - \zeta_i(\cos\gamma_i) - a\cos(\gamma_i \pm {}^{\pi}/_2)$ and $\rho_{yi} = y + h(\sin(\alpha + \gamma_i \pm {}^{\pi}/_2) - \zeta_i(\sin\gamma_i) - a\sin(\gamma_i \pm {}^{\pi}/_2)$. Expanding the norm in Eq. 1, one can define the terms e_{1i}, e_{2i} and e_{3i} as

$$\underbrace{(-2l_2\rho_{yi})}_{e_{1i}} \sin\theta_i + \underbrace{(-2l_2\rho_{xi})}_{e_{2i}} \cos\theta_i + \underbrace{\rho_{xi}^2 + \rho_{yi}^2 + l_2^2 - l_3^2}_{e_{3i}} = 0. \tag{2}$$

The transcendental Eq. 2 may be solved using the *Weierstrass substitution*:

$$\theta_i = 2\tan^{-1}\left(\frac{-e_{i1} \pm \sqrt{e_{i1}^2 + e_{i2}^2 - e_{i3}^2}}{e_{i3} - e_{i2}} \right). \tag{3}$$

The angles β_i can be found in a similar fashion [2]. The Jacobian matrix can be determined by taking the time-derivative of the geometrical constraint described by Eq. 2:

$$\begin{aligned} \dot{x}\underbrace{[l_3\cos\beta_i]}_{a_{i,1}} + \dot{y}\underbrace{[l_3\sin\beta_i]}_{a_{i,2}} + \dot{\alpha}\underbrace{[l_3\, h\sin(\beta_i - \gamma_i - \alpha \pm {}^{\pi}/_2)]}_{a_{i,3}} = \\ = \dot{\theta}_i\underbrace{[l_2\, l_3\sin(\beta_i - \theta_i)]}_{b_{i,i}} + \dot{\zeta}_i\underbrace{[l_3\cos(\beta_i - \gamma_i)]}_{b_{i,i+3}}. \end{aligned} \tag{4}$$

The Jacobian matrix, $\mathbf{J}$, can be found by the matricial relation:

$$\dot{\mathbf{X}} = \mathbf{A}^{-1}\mathbf{B}\dot{\boldsymbol{\Theta}} = \mathbf{J}\dot{\boldsymbol{\Theta}}. \tag{5}$$

where $\mathbf{A} = diag(a_{i,j}) \in \mathbb{R}^{3\times3}$. For the non-redundant 3$\underline{\text{R}}$RR manipulator, $\mathbf{B} = diag(b_{i,i}) \in \mathbb{R}^{3\times3}$. For the redundant manipulator 3$\underline{\text{PR}}$RR, $\mathbf{B} = [diag(b_{i,i})\; diag(b_{i,i+3})] \in \mathbb{R}^{3\times6}$. Several kind of singularities can occur if the matrices $\mathbf{A}$ or $\mathbf{B}$ are not full rank. In a similar way, the Jacobian matrices, denoted as $\mathbf{K}$ in this work, that relate the active joints' inputs with the time derivative of all moving parts' posture can be found by deriving kinematic constraints [2].

2.3 Dynamics

The equations of motion of the manipulators under study are briefly described in this section. The derivation of these equations using the Newton-Euler formulation can be found in [9]. The relation between the generalized forces, $\boldsymbol{\tau_g} \in \mathbb{R}^{(3)\times1}$ for the the non-redundant 3$\underline{\text{R}}$RR manipulator and $\boldsymbol{\tau_g} \in \mathbb{R}^{(3+M)\times1}$ for the the redundant 3$\underline{\text{PR}}$RR manipulator, applied by the actuators and the forces and moments on the system can be expressed by:

$$\boldsymbol{\tau_g} = \mathbf{M}\ddot{\boldsymbol{\Theta}} + \mathbf{V}\dot{\boldsymbol{\Theta}}, \tag{6}$$

where

$$\mathbf{M} = \mathbf{J}^T\mathbf{Z}_e\mathbf{J} + \sum_{i=1}^{3}\sum_{j=0 \text{ or } 1}^{2} \mathbf{K}_{ij}^T\mathbf{Z}_{ij}\mathbf{K}_{ij} \quad \text{and} \tag{7}$$

$$\mathbf{V} = \mathbf{J}^T\mathbf{Z}_e\dot{\mathbf{J}} + \sum_{i=1}^{3}\sum_{j=0 \text{ or } 1}^{2} \mathbf{K}_{ij}^T\mathbf{Z}_{ij}\dot{\mathbf{K}}_{ij} + \sum_{i=1}^{3}\sum_{j=0 \text{ or } 1}^{2} \mathbf{K}_{ij}^T\mathbf{N}_{ij}\mathbf{K}_{ij}; \tag{8}$$

where j indicates the moving part ($j = 0$ indicates the redundant actuator presented in the redundant manipulator, $j = 1$ the link 1 and $j = 2$ the link 2). The matrices $\mathbf{Z}_{ij}$, $\mathbf{N}_{ij}$ and $\mathbf{Z}_e$ are defined as:

$$\mathbf{Z}_{ij} = \begin{bmatrix} m_j & 0 & -m_j s_{ij}\sin\phi_{ij} \\ 0 & m_j & m_j s_{ij}\cos\phi_{ij} \\ -m_j s_{ij}\sin\phi_{ij} & m_j s_{ij}\cos\phi_{ij} & I_j \end{bmatrix}, \tag{9}$$

$$\mathbf{N}_{ij} = \begin{bmatrix} 0 & 0 & -m_j\dot{\phi}_{ij}s_{ij}\cos\phi_{ij} \\ 0 & 0 & -m_j\dot{\phi}_{ij}s_{ij}\sin\phi_{ij} \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{Z}_e = \begin{bmatrix} m_e & 0 & 0 \\ 0 & m_e & 0 \\ 0 & 0 & I_e \end{bmatrix}, \tag{10}$$

where the mass and inertia of the end-effector are m_e and I_e, the mass and the inertia of the moving part j of the kinematic chain i are m_{ij} and I_{ij}, s_{ij} is the distance between of the mass center of the moving part and its pivotal point, ϕ_{ij} is the angular position of the moving part.

A force-torque transformation, $\mathbf{T} = diag(0, 0, 0, K/(2\pi), K/(2\pi), K/(2\pi))$, should be used to the last three terms of the redundant inputs yielding the required torques for executing a predefined trajectory. In this way, $\boldsymbol{\tau} = \mathbf{T}\boldsymbol{\tau_g}$, where K is the lead of the linear guide.

3 Metrics

The redundancy resolution schemes are proposed based on metrics for evaluating the performance of parallel manipulators. In this work, the metrics described in this section should be maximized.

Condition Number of the normalized matrix A: Regions near to singular positions present ill-conditioned Jacobian matrices. In this way, a good strategy to avoid such regions is to evaluate the condition number of the Jacobian matrix. In this work, the condition number of the matrix $\mathbf{A}$ is used. Due to the presence of different units, this matrix should be normalized [12]:

$$\mathbf{A}' = \begin{bmatrix} a_{11} & a_{12} & a_{13}/L_c \\ a_{21} & a_{22} & a_{23}/L_c \\ a_{31} & a_{32} & a_{33}/L_c \end{bmatrix}, \tag{11}$$

where $L_c = \sqrt{2}\,h$ is the normalized length. The first metric, to be maximized, is defined as:

$$\phi_1 = -\kappa(\mathbf{A}') = -\frac{\sigma_{max}(\mathbf{A}')}{\sigma_{min}(\mathbf{A}')} \tag{12}$$

Distance between A_i and C_i: The limits of the manipulator's workspace should also be considered for a successful redundancy resolution scheme. If the links are perpendicular, this condition is avoided. Considering $l_t = \sqrt{l_2^2 + l_3^2}$ and keeping the position vector $\|\mathbf{r}_{\mathbf{A_iC_i}}\|$ between the upper and the lower limits is the strategy adopted in this work. This can be mathematically described by:

$$\phi_2 = -\sum_{i=1}^{3} \left[\left(l_t - \|\mathbf{r}_{\mathbf{AiC_i}}\|\right)^2 + \left(\frac{1}{\|\mathbf{r}_{\mathbf{A_iC_i}}\| - |l_2 - l_3|} + \frac{1}{l_2 + l_3 - \|\mathbf{r}_{\mathbf{A_iC_i}}\|} \right) \right]. \tag{13}$$

Limiting the amplitude of $\boldsymbol{\zeta}$: The limitation on the redundant actuators' position ζ_i for each kinematic chain i should be taken into account. This can be done by the following function:

$$\phi_3 = \sum_{i=1}^{3} \left(\frac{1}{\zeta_i - \zeta_{max}} - \frac{1}{\zeta_i + \zeta_{max}} \right). \tag{14}$$

where ζ_{min} and ζ_{max} are the lower and the upper bound of ζ_i.

Guaranteeing a minimum distance between the end-effector and the linear guides: The limitation on the redundant actuators' position ζ_i can be imposed for guaranteeing a minimum distance between the end-effector and the linear guides. This distance can be found for the geometrical relations and is denoted by ζ_s. The metric can be described by:

$$\phi_4 = -\sum_{i=1}^{3} \left(\frac{1}{|\zeta_i - \zeta_s|} \right). \tag{15}$$

where ζ_{min} and ζ_{max} are the lower and the upper bound of ζ_i.

Dynamic performance (index 1): By imposing a null velocity to the system, the relation between the end-effector's acceleration and the actuator's inputs can be calculated by $\boldsymbol{\tau} = \mathbf{TMJ}^{-1}\ddot{\mathbf{X}} = \mathbf{J}_d\ddot{\mathbf{X}}$. A metric can be derived by the evaluation of the lowest singular value of the matrix $\mathbf{J}_d^{-1}$:

$$\phi_5 = \sigma_{min}(\mathbf{J}_d^{-1}). \tag{16}$$

Dynamic performance (index 2): For global redundancy resolution scheme, the task to be executed is know. In this way, the maximum required torque to perform this predefined task can be also exploited for evaluating the dynamic performance. In this way,

$$\phi_6 = -||\boldsymbol{\tau}||_\infty. \tag{17}$$

4 Redundancy Resolution Schemes

The weighted cost function to be exploited by both redundancy resolution scheme is given by:

$$\phi = \sum_{k=1}^{6} c_k \phi_k, \tag{18}$$

where c_k are the weighting factors.

4.1 Direct Global Strategy

For a predefined task, the inputs of the redundant actuators can be calculated by solving a optimization problem. These inputs are mathematically formulated as a fifth order polynomial function. In this way, this optimization problem should seek for the initial and final values, $\zeta^I_{1,2,3}$ and $\zeta^F_{1,2,3}$, respectively. The optimization can be posed by the following problem:

$$\begin{gathered} max\ _{\zeta^I_{1,2,3},\zeta^F_{1,2,3}}\ \phi \\ \text{subject to} : \zeta_{min} \leq \zeta^{I,F}_{1,2,3} \leq \zeta_{max}. \end{gathered} \tag{19}$$

4.2 Two-Stage Global Strategy

For a predefined task, the inputs of the redundant actuators can be calculated by solving a two-stage optimization problem. Firstly, ideal inputs for the redundant actuators are calculated by integrating numerically the local redundancy resolution scheme described by a problem involving the gradient of a cost function:

$$\dot{\boldsymbol{\zeta}} = c_H \nabla\phi(\boldsymbol{\zeta}), \tag{20}$$

where c_H should be selected in order to guarantee the numerical integration. This numerical integration yields reference inputs for the redundant actuators,

$\boldsymbol{\zeta}(k)^R$, discretized in time. These inputs may require unrealistic accelerations. The second stage, is mathematically described by [10]:

$$min\ _{\zeta_{1,2,3}(k)} c_r \sum_{i=1}^{3} \|\zeta_i(k)^R - \zeta_i(k)\|^2 + c_a \sum_{i=1}^{3} \|\mathbf{a}^T \mathbf{a}\| \tag{21}$$
$$\text{subject to} : \zeta_{min} \leq \zeta_{1,2,3}(k) \leq \zeta_{max},$$

where c_r and c_a are weighting factors, $\mathbf{a}$ are the acceleration vector obtained by second order finite approximation. This problem can be solved using Differential Dynamic Programming [11] yielding inputs for the redundant actuators.

5 Results

Details about the data values used in the numerical calculations are given in [10, 9]. Both redundancy resolution schemes are evaluated for the derivation of the redundant actuators' inputs for a pick-and-place task illustrated in Fig. 3(a) that should be executed in 1.3s. The non-redundant manipulator 3RRR is unable to perform this task, as illustrated in Fig. 3(b), since it crosses a singular region.

Both redundancy resolution schemes evaluated in this work were capable to deliver inputs to execute the trajectory in a satisfactory manner considering $c_1 = 0.02$, $c_2 = 0.95$, $c_3 = 0.01$, $c_4 = 0.005$, $c_5 = 0.01$, $c_6 = 0.005$, $c_H = 25$, $c_r = 1$ and $c_a = 0.003$. Figures 4 and 5 show the required currents for the execution of the task. The Direct Global Strategy yielded the same initial and final inputs for the redundant actuators. In other words, the redundant actuators are moved before the execution of the task and are kept still during the task. In this way, only the required currents of the non-redundant actuators are showed in Fig. 4. The Direct Global Strategy required smaller currents for executing the task compared to the Two Stage Global Strategy when the same weights are

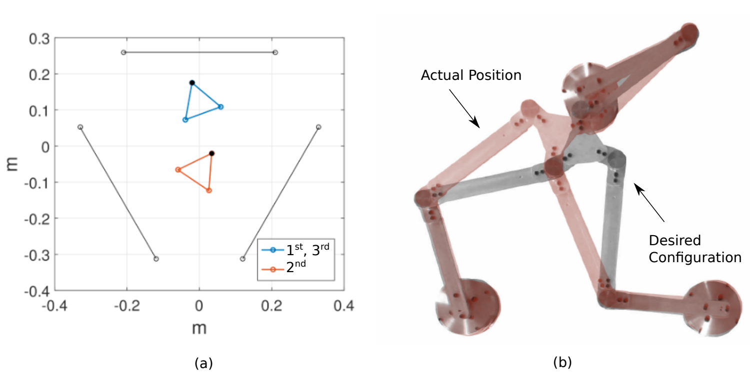

Fig. 3. (a) Task to be executed and (b) Actual and desired configurations for the 3RRR

used. Nevertheless, a conclusion can only be drawn if a complete evaluation of the selection of these weights is carried on.

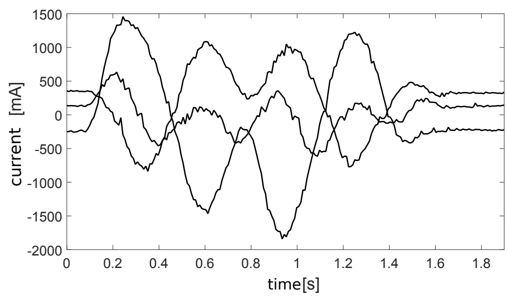

Fig. 4. Non-redundant actuators' currents using the Direct Global Strategy

6 Conclusions

In this work, two redundancy resolution strategies are exploited for deriving the inputs of the redundant actuators of a 3$\underline{PR}$RR planar parallel manipulator for a tracking trajectory problem. The Direct Global Strategy is a direct global one that attempts to minimize a weighted cost function. The Two Stage Global Strategy attempts to find reference inputs and by exploiting the Differential Dynamic Programming imposes limitations on the accelerations. When the same weights are used for both strategies the direct one required smaller currents than the other strategy. This result is dependent on the task and the weights.

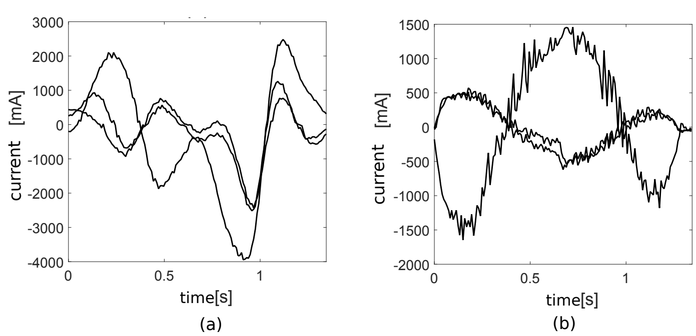

Fig. 5. Currents of the (a) non-redundant and (b) redundant actuators using the Two-Stage Global Strategy

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