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Optimal Dimensional Design of Parallel Manipulators with an Illustrative Case Study: A Review

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ABSTRACT

Optimal dimensioning is a fundamental problem in the design of Parallel Manipulators (PMs). However, this problem is revealed to be very hard to solve because many PM performance criteria are antagonistic. In addition, the required technical specifications can be difficult to obtain because they are usually dependent on the end-effector posture and the task to be performed. In this paper, we present an overview of the different approaches used in the optimal design of PMs, as well as the main difficulties encountered. Technical solutions are proposed to solve the problem of optimal dimensional design of PMs. In addition, a seven-stage optimal design methodology for PMs is proposed. Finally, we present an illustrative application of the methodology developed for a 5R parallel manipulator with two degrees of freedom.

1. Introduction

Among the flaws that characterise serial manipulators, poor dynamic performance, low stiffness, and the structure makes them sensitive to bending at high loads and vibrating at high speeds. As a result, these flaws lead to a lack of precision and stability. The need to overcome these drawbacks and improve the dynamic performance of serial robots (velocities, accelerations, etc.) either improves the components of these robots [1] or drastically changes their architecture. In this context, Parallel Manipulators (PMs) constitute an interesting and proven alternative, as they are designed on the basis of parallel mechanism architecture.

Indeed, PMs are renowned for their high performance, very high dynamics, rigidity, and precision [2–5] and are constantly replacing serial robots. This becomes particularly the case in precision industries, such as machining, where parallel kinematic machine tools are developed. Mechanisms with parallel structures have also been proposed and studied for the design and development of a few prototypes to solve problems related to the domain of high-speed machining (HSM), where the need for precision and high rigidity is crucial [3]. These machines-tools, with parallel kinematics, make it possible to obtain better quality machining with a cycle time significantly shorter than one with a serial structure. Another field of application of PMs is the packaging industry, particularly pick-and-place tasks (which consists of transporting an object from one point to another at high speed), where the dynamics (speeds and accelerations) are crucial. For this type of application, PMs can achieve accelerations of several tens of g ($\leq 10g$) [6].

Indeed, PMs have piqued the interest of researchers and industry because they offer a good alternative solution to the problems typified by their serial counterparts, which are penalised by the high mass of their moving elements. Each motorised axis supports the next axis, which gives rise to bending problems that may affect their accuracy. However, PMs have the effective potential to offer high precision and good dynamic performance.

Nevertheless, the optimal dimensional synthesis of PMs, which aims to determine the dimensions of a given topology, remains an essential objective in the optimal design of PMs, since the performance criteria of a given structure depend very strongly on the dimensions of its geometric parameters (dimensions of the segments, strokes of the actuated links, points of attachment of the links connecting the mobile platform to the fixed base, etc.) which need to be optimised. This task remains a difficult and open problem because of the many quantitative and, sometimes, qualitative criteria

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that can interfere in a complex way and be antagonistic (kinematic performance, workspace, avoidance of the vicinity of singularities, dynamic performance, etc.).
Exploring and finding methods and techniques that make it possible to perform an optimal dimensional synthesis in a multi-objective context is, therefore, a challenge which is of paramount importance, both scientifically and technologically. This optimal design must face the problems related to these PMs, such as the small workspace, the non-homogeneity of kinematic and dynamic performance within the workspace (due to the presence of singularities), the non-linearity between operational and joint speeds, etc.; their performance would be very sensitive to their dimensions (geometric parameters). This now requires the proposition of a dimensional design methodology that aims at improving the performance of PMs (fully parallel manipulators) and takes into account the different requirements and constraints.
The main contributions of the present paper are as follows:

- Presenting and critiquing the major literature methods used in the dimensioning of the PMs.
- Outlining the major issues encountered and the solutions typically used during the optimal design analysis of PMs.
- Proposing an optimal overall design methodology consisting of seven steps. The advantage of the proposed methodology is that it enables us to simultaneously take into account and analyse several performance criteria with a direct impact on the dimensional synthesis of PMs by using the concept of a multi-objective optimisation method. Moreover, it even allows the choice of the final optimal solution (a single optimised vector of the geometric parameters). In addition, a tolerance synthesis of the optimised geometric parameters is also proposed, in order to guarantee that the PM keeps its performance criteria despite uncertainties in the various parameters and operating conditions.

The rest of this paper is organised as follows: in Section 2 we present the main approaches used, usually dedicated to optimally designing PMs. In the third section, we present the main difficulties encountered during the dimensional design of PMs, as well as the available technical solutions. Finally, we present a global methodology as a proposal for optimally designing PMs (geometric dimensioning). The paper is concluded by a conclusion and intended future work.

2. Main approaches used in the dimensional design of PMs

2.1. Performance chart

Performance chart (also called Atlas method) is widely used in classical (conventional) design and in most design manuals. It is a performance graph that can show the relationship between a performance index and the associated design parameters in a limited space, in the form of an Atlas. Then, these performance charts can show how the considered criteria are antagonistic [7]. After this stage, the designer can use these atlases to choose the design parameters according to the specifications requirements. This method was used by several researchers to dimension the PMs. In their work, Feng Gao et al. [8] used the geometric model to obtain the analytical relationships between the link lengths of 2-DOF (Degree-Of-Freedom) parallel planar manipulators (5 geometric design parameters) and their performance criteria (2 criteria), based on the global condition number and global velocity indices. The model is used to develop graphs for the analysis and design of the mechanisms. Finding the optimal geometric parameters from the atlases representing the performance measures that describe the overall behaviour of the manipulator is a very complicated operation, even with a limited number of parameters and criteria.

This was also explored in the prior studies by Liu et al. [7]. The authors proposed an optimal kinematic design methodology based on performance charts (Atlas), for parallel mechanisms with fewer than five design parameters. According to the authors, the strength of this methodology is that a performance criterion corresponding to a graph can globally represent the relationship between the criterion and the design parameters. They note that certain stages in this methodology are also useful for optimal design based on the cost function. The results obtained can be used to develop a computer-aided design system for parallel mechanisms. The proposed design methodology can also be generalised for serial and parallel manipulators, or for any other mechanism, but whether the number of criteria increases with an antagonism between these criteria can easily be seen and, obtaining an optimal vector of the geometric parameters will become a very complicated operation. Puglisi et al. [9] used the performance chart method for the dimensional synthesis of a 3PSU-1S spherical parallel manipulator in order to have the best performance, in terms of workspace, dexterity, and isotropy. They proposed a new bounded index, called the global level and distribution ratio index. Then they analysed the relationship between this index and the geometric parameters. The optimal geometric vector was obtained from the analysis of graphs relating the performance index and the geometric parameters. A prototype showing the best performance, in terms of the composite index, was presented. In this case, use of a single performance index lead to satisfactory results because the number of geometric parameters remained lower. Many studies and works have used this method to determine the geometric parameters of designed PMs [10–12].

2.2. Critique of the performance charts

- The application of this method is not possible if the number of geometric parameters is large, hence the need to minimise the number of these parameters and know how to do it without influencing the design. It is clear that this technique remains very limited and can only give satisfactory results if the numbers of geometric parameters and performance criteria are small.
- It is difficult to define the range of variation of geometric parameters in a reasonable way, since each parameter of a manipulator can have any value between zero and infinity. Therefore, it is almost impossible to illustrate a graph in infinite space.
- It is difficult to achieve all optimal results.
- The most annoying thing about this method is that the optimal result is fuzzy.

2.3. Objective function (also called cost function)

The most widespread optimisation technique that has been the subject of seminal contributions in several works [13–15] consists of the use of objective functions. This technique is either based on a single-objective optimisation (only taking into account one objective among several, such as workspace, kinematic performance, dynamic performance, stiffness, etc.) or a multi-objective optimisation (all objectives are taken into account, simultaneously). This method has been used by several researchers for the optimal design of PMs [16–18]. It is based on maximising (minimising) one or more cost (objective) functions. Mathematically, the problem can be formulated as follows:

Find a vector $P^* = [P_1^*, P_2^*, ..., P_n^*]^T$
that: **Minimize** $F(P) = [f_1(P), f_2(P), ..., f_k(P)]^T$
with $g_m(P) \leq 0$ (m inequality constraints)
and $h_l(P) = 0$ (l equality constraints)
$P^* \in \mathbb{R}^n$: Vector of the decision variables
$F(P^*) \in \mathbb{R}^k$: Vector of the objectives function

2.3.1. Dimensioning of PMs based on consideration of a single performance criterion (single-objective optimisation)

To apply this technique, it is necessary to complete the geometrical, kinematic, and dynamic modelling. Then, the different performance criteria of the PMs are determined, which will subsequently be used as objective functions. It should be noted that the modelling phase is essential because the specifications of the manipulator must be respected, taking into account the active (actuator displacements) and passive links, workspace, desired precision, maximum desired velocities and accelerations of the end-effector relative to those of the actuators, maximum forces on each leg of the manipulator (as well as on the end-effector throughout the workspace), and the homogenisation of performance within this workspace.
The main performance criteria (objectives) used in the dimensional design of PMs can be defined as:

i) **Workspace**
The literature review demonstrated that workspace is a fundamental and necessary criterion to consider when designing PMs [18–23] because PMs frequently have a smaller workspace than serial manipulators. It is noteworthy that their characterisation (area and shape) remains a challenging problem because of the complexity of the direct kinematics.
The workspace is represented by all the situations in space that the end-effector can reach (configurations accessible by the end-effector). This space is defined by the limits imposed by the articular movements (active and passive), the lengths of the segments and the internal collisions [18]. According to Merlet [22], different types of

workspace exist for PMs. More generally, and from a mathematical standpoint, the workspace [20] is defined by the application f of the articular space (Q) of the n dimension in the operational space (X) of dimension $m \leq 6$:

$$q \in (Q) \rightarrow X = H(q) \in (X) \tag{1}$$

The domain (Q) is admissible for the articular variables and is, generally, a hyper parallelepiped, defined by:

$$Q = \{q \in (EA), \forall j \leq c, (q_j)_{min} \leq q_j \leq (q_j)_{max}, j = 1, \cdots, n\} \tag{2}$$

where $(q_j)_{min}$ and $((q_j)_{max}$ are the stops on the A_j joint. It is assumed that there are no obstacles in the workspace. A large number of existing studies in the broader literature have examined this research axis. Gosselin and Angeles [24] analysed the optimal design of a 3-DOF planar manipulator. Their objectives were the maximisation of the global workspace and obtaining an isotropic Jacobian matrix. In the same context, Roger Boudreau et al. [25] presented a method using a genetic algorithm to obtain 26 architectural parameters of a Gough-Stewart platform, to have an attainable space as close as possible to a previously prescribed space. Ottaviano and Caccarelli [26] optimised the design of a PM called CaPaMan by using the characteristics of the workspace. They simplified the complexity of evaluating the workspace by using an approximate parallelepiped.
The work by Merlet [19] presented an algorithm to determine all the possible geometries of Gough type 6-DOF parallel manipulators whose workspace must include a desired workspace. This algorithm takes into account the length limits of the segments, the mechanical limits on the passive joints, and the interference between links. Zhe Wang et al. [27] presented an algorithm to determine the workspace of a parallel machine tool and facilitate the machining operations of mechanical parts.
Perdro et al. [28] presented a new Monte Carlo method (called the Gaussian Growth method) to calculate the workspace of PMs. This method focuses on filling and improving the accuracy of poorly defined regions of the workspace. It generates an inaccurate original workspace using a classic Monte Carlo method, then densifies and expands that original workspace using a Gaussian distribution until the workspace boundaries are reached.
Aboulissane et al. [29] presented an algorithm, implemented in the CATIA software, for determining the workspace of the Delta parallel robot by showing the impact of different design parameters on the accessible workspace of the DELTA manipulator robot. To do this, a method based on an optimisation criterion, which is the conditioning of the Jacobian matrix, is used to keep only the points guaranteeing a minimum value of this criterion. In other words, only the accessible points with a number of conditioning below an imposed threshold are retained to represent the sub-workspace. This procedure allows for a dexterous workspace W_{dextre} (the set of positions of the reference point of the end-effector for which all orientations are allowed [22]).
Hence, the following objective function was obtained:

$$W_{dextre} \rightarrow max \tag{3}$$

In [30], the authors proposed a methodology for the optimal design of the Stewart platform. This methodology is based on the use of the concept of single-objective optimization. They used neural networks to calculate the direct geometric model of Stewart's platform and genetic algorithms to estimate the robot's workload. Then, they used genetic algorithms to dimension the structure of the robot by maximising the accessible workspace.

ii) **Kinematic performance**
This criterion measures the ability of the PM to arbitrarily change its position and orientation or to apply forces and torques in an arbitrary direction in all directions of the workspace [31, 18]. These kinematic performances are closely linked to the relationship between operational and joint velocities (by virtue of the principle of displacement/effort duality) and the ratio between the static forces applied to the end-effector and the torques measured on the actuators. They measure:

- The precision with which the movement of the end-effector and the contact forces can be controlled by the forces and movements of the joints,
- The proximity of a unique configuration.

These performances can be evaluated by the following relationships:

$$\begin{cases} \dot{X} = J\dot{q} \\ \tau = J^T f \end{cases} \tag{4}$$

$\dot{X}$: Operational velocities; $\dot{q}$: Joints velocities; f : Generalised forces; τ: Joint forces. The Jacobian inverse matrix J^{-1} is used to measure these performances.
The conditioning index is a global index that can evaluate the dexterity, isotropy, and static stiffness of the parallel manipulator [32–35]. This has been discussed by a great number of authors in the literature [36–41]. They used the condition number of the Jacobian matrix $cond(J^{-1})$ as an optimisation criterion for the optimal design of PM. When the condition number is 1, the configuration is called isotropic. This index was first introduced by Yoshikawa [42] for serial manipulators, and then applied to PM [36, 43, 44]. Pittens et al. [45] used this index to study several configurations of Stewart's platform, to optimise local dexterity and determine the optimal architecture and effective workspace. On the whole, and in the accessible workspace, this index is defined by:

$$\eta_J = \frac{\int_w \kappa_j dw}{dw} \tag{5}$$

With: $\kappa_j = \|J^{-1}\|.\|J\| = \frac{\sigma_{max}(J^{-1})}{\sigma_{min}(J^{-1})} \in [1, \infty[$, $\sigma_{max}(J^{-1})$ and $\sigma_{min}(J^{-1})$ are respectively the largest and small singular value of the Jacobian inverse matrix $\|J^{-1}$.
$\|J^{-1}\|$: the Euclidean norm of J^{-1}.

iii) **Dynamic performance**
As in many non-Cartesian manipulators, in the case of this PM, the nonlinearity and coupling of the equations of motion makes them more difficult to control. However, with current technological means, there are no obstacles. The variation in inertia makes the choice of actuators and the adjustment of the control gains more complex, when we are looking to achieve high acceleration. In order to increase dynamic performance, inertia must be reduced to a minimum. In the present case (dimensional synthesis), we are only interested in optimising the inertia matrix, which is an important criterion for increasing the dynamic capacities of PMs.
Consequently, by 'dynamic performance', we mean the dynamic dexterity essential for the planning of trajectories [46]. It is best when, from any point in the workspace, there is easy movement of the end-effector (greater acceleration capacity) along all directions in the workspace. Some dynamic dexterity evaluation measures, such as dynamic manipulability [47], have been proposed to measure dynamic performance, namely:

- ***Dynamic isotropy index***
  The inverse dynamic model of a parallel manipulator, generally given by:

$$\Gamma = I(q)\ddot{q} + V(q, \dot{q} + G(q) \tag{6}$$

Such as: $\Gamma = [\Gamma_1, \Gamma_2 \ldots \Gamma_n]^T$: *Vector of the generalized torques (forces)*

$q(t) = [q_1(t), q_2(t) \ldots q_n(t)]^T$: *Vector of the joint position variables*
$\dot{q}(t) = [\dot{q}_1(t), \dot{q}_2(t) \ldots \dot{q}_n(t)]^T$: *Vector of the joint velocities*
$\ddot{q}(t) = [\ddot{q}_1(t), \ddot{q}_2(t) \ldots \ddot{q}_n(t)]^T$: *Vector of the joint accelerations*
$I(q)$: *Inertia matrix of the robot*
$V(q, \dot{q})$: *The centrifugal and Coriolis term,*
$G(q)$: *The gravitational term.*

The condition number of the mass matrix ($M = J^{-T} I(q) J^{-1}$), denoted κ_M is used to measure the index, is defined by:

$$\kappa_M = \frac{\sigma_{max}(M)}{\sigma_{min}(M)} \tag{7}$$

where $\kappa_M = 1$ are called isotropic configurations. The objective function is defined as follows:

$$\kappa_M \rightarrow min \tag{8}$$

To describe the dynamic dexterity on the whole accessible workspace by the end-effector, we used the global dynamic dexterity index η_M [36, 48], which is defined by:

$$\eta_M = \frac{\int_W \kappa_M dW}{\int_W dW} \tag{9}$$

Asada et al. [49, 50] proposed to evaluate the dynamic performance through the construction of a Generalised Inertia Ellipsoid, which indicates that the movement of the end-effector is easy (high acceleration) in the direction of the main (large) axis of this ellipsoid and more difficult (less accelerated) in the direction of the main (small) axis. In the case where the length of the two axes is the same, the performance of the accelerations is isotropic, which enhances the dynamic dexterity. Tadokoro et al. [46] proposed a measure of stochastic dynamic manipulability, which assesses dynamic dexterity by considering the deviation of the direction of end-effector acceleration. Other work was carried out by several researchers who saw that, to improve the dynamic performance of a PM, it is necessary to take into account the dynamics. MA and Angeles [51] proposed the notion of the isotropy of the dynamics, to determine the dimensions and the inertial parameters of a 3-DOF manipulator. Jun Wu et al. [52] used the condition number in the inertia matrix to evaluate the dynamic dexterity of a 2-DOF plane parallel manipulator.

iv) **Stiffness**

Stiffness is defined as the resistance to elastic deformation in a static or dynamic regime (such as PMs, subassemblies or elements), under the effect of external forces [39]. The reciprocal notion is complacency or flexibility. The stiffness value evolves according to the geometry and topology of the structure, and the position and orientation of the end-effector within the workspace. The stiffness of a PM at a given point in its workspace can be characterised by its stiffness matrix. This matrix combines the forces and torques applied to the end-effector. If we denote by $\sigma_{min}(S)$ the minimum singular value of the stiffness matrix S and the condition number byκ_S, the objective function is given by:

$$\kappa_S \rightarrow min \tag{10}$$

To characterise it on the whole accessible workspace by the end-effector, we use the index η_S, which is defined by:

$$\eta_S = \frac{\int_W \kappa_S dW}{\int_W dW} \rightarrow min \tag{11}$$

Numerous other studies focus on improving the stiffness of PMs as a performance criterion [43, 53–57].

v) **Energy consumption**

In [58], the authors have considered that the energy efficiency of a 4-degree-of-freedom symmetric parallel manipulator refers to its ability to perform the desired task while consuming the minimum amount of energy. This was an important design objective, as it can have a significant impact on the operating costs and an environmental impact of the manipulator. The goal was to simultaneously maximise the manipulator's workspace and dexterity while minimising its energy consumption. To evaluate the energy efficiency of the manipulator, the authors used a simplified model of its power consumption, based on the mechanical work required to move the end effector through its workspace. The power consumption model takes into account the effects of gravity, friction, and other factors that may affect the energy consumption of the manipulator. The authors present the results of their optimisation analysis for the case study of a 4-degree-of-freedom symmetric parallel manipulator. Furthermore, they show that the optimised design of the manipulator consumes less energy than the original design while still maintaining its workspace and dexterity.

In [59], the authors proposed the dynamic modelling and power optimisation of a 4RPSP+PS parallel flight simulator machine. The aim of this study was to develop a mathematical model that can accurately represent the behaviour of the simulator and optimise its power consumption. To optimise the power consumption of the machine, the author proposed a control algorithm that uses an adaptive fuzzy sliding mode controller (AFSMC) and a genetic algorithm (GA) to tune the controller parameters. The proposed AFSMC was used to regulate the position and velocity of the simulator, while the GA has used to find the optimal control parameters that minimise the power consumption. The results of the study show that the proposed mathematical model accurately represents the behaviour of the simulator, and the optimisation algorithm successfully reduces the power consumption by up to 15%. The author concluded that the proposed method can be applied to other parallel manipulators, and can contribute to the development of more efficient and sustainable flight simulators.

The context of the problem of minimising the electrical energy consumed by the robot manipulator can be expressed as follows:

"What is the best dimensioning of a robot manipulator (vector of optimised geometric parameters P^*) that allows the robot to consume the minimum amount of electrical energy while satisfying the constraints related to the task, to the robot, and to the environment?".
It is worth to note that the minimisation of the electrical energy absorbed by a robot manipulator can be obtained by considering:

- The task: by minimising the duration (T) of its execution and optimising the planning and localization of trajectories in the PM workspace.
- The choice and sizing of robot actuators.

Knowing that the electrical energy (E_{total}) emitted by a robot manipulator is given by:

$$E_{total} = \sum_{i=1}^{n} E_{a_i} = \sum_{i=1}^{n} P_{a_i}.T \tag{12}$$

where E_{a_i} denotes the energy absorbed by each actuator; n, the number of actuators in the robot; P_{a_i}, the electrical power absorbed by an actuator; and T, the task execution time.
and if the actuators are DC Brushless motors:

$$P_a = P_{other} + P_{mec} \tag{13}$$

where P_{other} represents the loss of power inside the motor (including armature, inductor, loss of iron: ferromagnetic materials). and, P_{mec} denotes the power transformed into mechanical form along with the mechanical losses due to the friction in transmissions and joints, ventilation, and vibration.
Hence, the energy consumed E_a by one actuator can be formulated as follows:

$$E_a = (P_{other} + P_{mec}).T \tag{14}$$

In other words, the total energy is the sum of the energies including the magnetic energy stored in the windings of the machine, the kinetic energy of the rotating masses, the energy dissipated by the losses in the windings, the energy dissipated by the friction of the rotor, and the energy supplied to the mechanical load.
The electrical power transformed into mechanical power can be described by the following expression:

$$P_{mec} = \tau.\omega \tag{15}$$

where τ denotes the motor torque, and ω the rotation speed.
It can be seen that the expression of the electrical energy consumed by the actuator depends on the motor torque τ, hence the objective function can be formulated as follows:

$$E_{total} \rightarrow min \Leftrightarrow \sum_{i=1}^{n} \tau_i \rightarrow min \tag{16}$$

2.3.2. Dimensioning of PMs based on simultaneous consideration of multiple performance criteria

Several studies suggest using this technique to size the PMs. For example, Pritschow [60] proposed a systematic method in six steps, for the design of a Hexaglid or Hexapod Parallel Kinematic Machine. Company et al. [3] proposed a global method for developing a UraneSX machine for three-axis machining operations (drilling and taping for the automotive industry). Stock et al. [61] proposed a method in the form of a multi-objective optimisation problem by the linear combination of the two indices based on manipulability and workspace, for the dimensioning of a linear Delta PM. Valasek [62] deconstructed the process of the optimal design of a structure and its geometric parameters into three levels by using computational tools capable of calculating mechanical properties on the entire accessible workspace. To optimise the Hexapod machine in machining operations, Kubler et al. [63] proposed taking into account the workspace, stiffness, and flexibility of the machine, all of which are important for machining tools. In their work, Merlet et al. [17] proposed an optimal design methodology based on interval analysis, to determine geometric parameters by meeting two performance criteria: workspace and precision. The major drawback of this technique is that it is very time-consuming.

In their paper, Neugebauer et al. [64] proposed a method that takes into account the kinematic and dynamic properties of the Delta robot. In [65] Ganesh et al. proposed the use of typical non-dimensional performance indices, namely the workspace volume index (WVI), the global translation stiffness index (GTSI) and the global rotation stiffness index (GRSI), for dimensioning PMs with 3-DOF. Bounab [66] proposed optimising the dimensional synthesis of the DELTA parallel mechanism by considering its kineto-elastostatic performance, as well as obtaining a maximum regular workspace by using evolutionary genetic algorithms to find all the possible compromises between several cost functions that conflict with each other. For the dimensioning of a 3-UPR manipulator (U: Universal joint, P: prismatic joint, and R: revolute joint), Matteo Russo et al. [67] proposed the volume of the workspace, the manipulator dexterity, static efficiency, and stiffness as objective functions for determining the geometric parameters of the mechanism. In [68], the authors provide a detailed review of the existing literature on the performance optimisation of parallel manipulators (PM). They cover a wide range of topics, such as workspace optimisation, stiffness optimisation, dynamic performance optimisation, and control optimisation. For each topic, the authors provide a summary of the research that has been done, along with the advantages and disadvantages of the different approaches. They conclude with a discussion of the future research directions in the field of parallel manipulators and their performance optimisation. There are many other formulations that take into account other performance criteria, such as positioning error, isotropy, velocity, force, etc. [69–76, 15, 77–80].

2.4. Critique of the objective function approach

To solve the mathematical optimisation problem (Section 2.3):

2.4.1. Critique of the single-objective optimisation

- Single-objective optimisation offers a solution that satisfies a single performance criterion (a single objective function) to the detriment of other criteria that are sometimes not taken into account when formulating the optimisation problem. A designer can use this type of formulation if they are interested in a single performance criterion.

- Taking into account a single performance criterion may sometimes be insufficient to determine all the geometric parameters of the PM [15].

- The choice of a single performance criterion is a risky choice and is not sufficient for a dimensional synthesis of the PM because the performances are often antagonistic (e.g. workspace and kinematic performances).

- If metaheuristic techniques are used (simulated annealing method, taboo search method, colony algorithms, genetic algorithms, PSO, etc.), then the problems encountered with the classical methods are overcome. The main disadvantage of these techniques is that they are time-consuming, since certain performance criteria must be evaluated on the whole accessible workspace, which makes them expensive in computation time terms. This reduces the number of parameters by making assumptions that do not influence the geometry of the optimal manipulator. This remains a delicate operation.

- The single-objective optimisation technique provides a single solution (point solution or optimal vector) to the optimised objective function, while the multi-objective optimisation provides a set of optimal solutions. Other difficulties arise as a result, particularly the optimal solution, which requires additional analysis of decision aid. Furthermore, it should be noted that, in the single-objective optimisation, the initial solution considerably influences the speed of convergence towards the optimal solution (if it exists).

2.4.2. Critique of the multi-objective optimisation

- We cannot process the objectives successively because, in this case, the result favours extreme solutions.

- Taking into account a single performance criterion may sometimes be insufficient to determine all the geometric parameters of the PM [15].

- Objective functions do not have the same physical dimensions, which makes it very difficult to use the weighted-sum-of-objective-functions method to transform the multi-objective optimisation problem into a single-objective optimisation problem. In knowing that this method is applicable only if the realisable domain is convex, its main drawback is that it does not allow one to calculate the compromise surface (Pareto front) when the domain is not convex. In other words, some non-dominated solutions cannot be obtained, regardless of the coefficients, because they are enclosed in a concavity [81].

- If we use the compromise method (of inequality constraints [82]) by transforming the multi-objective problem into a single-objective optimisation problem, while keeping the priority objective and transforming the other objectives into unequal constraints, this passage requires perfect knowledge of the problem in order to define the values of the constraints because there is a risk of losing the best dimensioning of the PM (to lose the optimum).
- In the multi-objective optimisation, the problem arises in the choice of the ranges of variation of the geometric parameters, where a priori knowledge of the problem is strongly recommended to guarantee obtaining the optimal solutions.

3. The main difficulties encountered during the dimensional design of PMs

There are certain difficulties associated with PMs, which make their design by the aforementioned approaches a delicate task. Among these difficulties, are the following:

3.1. Difficulties related to modelling (modelling challenges)

The modelling of the architecture is an essential point for carrying out a geometric synthesis of the chosen architecture. Its purpose is to determine a system of equations relating the position and the orientation of the end-effector to the joint positions (kinematic model).
During the optimal design of a PM, one necessarily goes through the modelling phases (geometric, kinematic, and dynamic), in order to establish the relationship between its geometric parameters and its performance criteria. This transition is not always easy. Many problems are encountered and may include:

- Problems related to the number of geometric parameters;
- The kinematic model is sometimes difficult to obtain. To overcome this difficulty, different solutions have been proposed in the literature, such as Newton's method of resolution [44] and the interval analysis method [83];
- Non-linearity between kinematic and differential kinematic equations may lead to a great complexity of design.

3.2. Difficulties related to the workspace

- The particular architectures of PMs lead to smaller manipulator workspaces than those of their serial counterparts. This is due to the additional constraints imposed by the closed kinematic chains of these mechanisms. Therefore, they are characterised by a reduced (restricted) workspace. One possible solution to this problem consists of using linear actuators oriented in the same direction. Another solution is to use electric rotary actuators; by arranging them so that they browse the largest common area, we can obtain a large workspace [84].
- PMs can also be difficult to design [37] because the relationships between design parameters and the workspace, as well as the behaviour of the manipulator in the workspace, are by no means intuitive.
- low workspace/size ratio compared to serial manipulators.
- The presence of various singularities in the workspace, where the number of DOF of the PM may change instantly. It is important to determine these configurations because, in their proximity, joint efforts can tend towards infinity, causing a failure of the manipulator and/or leading to a loss of control of the end-effector or even to a deterioration of the mechanical structure of the PM. This presents the most critical point when designing a PM. To limit the problems of singularities, one of the solutions is to ensure that the workspace is dexterous (Section 2.3.1.1) and to develop a more complete model that includes all types of PM singularities: over-mobility, under-mobility, and internal (or constraint).
- There is difficulty in determining the workspace and its often irregular shape, which is embarrassing for planning appropriate trajectories. This is because the workspace of a PM cannot be decoupled into 3D workspaces characterising the possible translation and orientation movements. One solution consists of developing and perfecting numerical methods and algorithms for the determination of the workspace of various PMs. These algorithms take into account the range of actuators, the mechanical stops of the joints and the interference between the segments.

3.3. Difficulties related to the kinematics of PMs

- Highly variable coupling between the different kinematic chains; this feature often complicates the control of manipulators. To overcome this difficulty, oversizing the actuators may be a possible solution.

- There is a strong coupling between the movements of the different kinematic chains. As a result, even simple trajectory generation often requires perfectly coordinated action from all actuators. Fortunately, this difficulty no longer arises because, nowadays, electronic and computer resources have greatly developed.

- non-uniformity in force and velocity transmission, as well as stiffness characteristics within the workspace. This is due to the presence of singularities in the workspace.

- The elements that make up the Jacobian matrix $\boldsymbol{J}$ are of different units (related to positions and orientations). As a result, some performance indices no longer have a clear physical meaning. One relevant example can be the condition number, which is widely used in the evaluation of performance criteria.
  To overcome this problem, it would be necessary to homogenize the elements of the Jacobian matrix. To this end, several propositions have been made in the literature by several researchers [85]: Angeles [86] suggested using 'the characteristic length', i.e., by dividing the elements related to the positions by a characteristic length L_c. In [87], the authors presented a method for the kinetostatic optimisation of revolute-coupled planar manipulators. One of the key challenges in this optimisation is that the terms of the Jacobian matrix may have different units, making it difficult to compare and optimise different manipulators. To address this issue, the authors introduced the concept of a characteristic length, which is a length scale that is representative of the size of the manipulator. By scaling the Jacobian matrix by the characteristic length, the terms of the matrix can be put into a dimensionless form, allowing for easier comparison and optimisation.
  Stucco et al. [88, 89] proposed to use a scaling matrix, which involves multiplying a design matrix by diagonal scaling matrices, corresponding to the range of joint and task space variables before and after. The Jacobian normalization, according to the authors, leads to a practical interpretation of a robot's "characteristic length" as the desired ratio of maximum linear and angular force or velocity.
  Hosseini et al. [90] used a technique based on a weighting factor to optimize the workspace of the Tricept Parallel Manipulator. Mansouri et al. [91] proposed, by means of the power concept, a new kinetostatic performance index for robot manipulators. Another technique based on a point-based method is also widely used. The authors in [92] developed a new numerical formulation of the velocity equation that can be applied to any topology of spatial mechanisms. This approach is intended for general-purpose software in computational kinematics. It can be used to solve all Jacobian problems and check performance requirements at the design stage.
  In [93], a new formulation is proposed to avoid the problem of dimensional dependency. Two dexterity indices are presented for planar manipulators: the first one is based on a redundant formulation of the velocity equations; whereas, the second one is based on the minimum number of parameters.
  By using a systematic approach, Liu et al. in [94] formulated the dimensionally homogeneous Jacobian; the proposed approach is general for f-DOF (with $f \leq 6$) parallel manipulators having coupled translational and rotational motion capabilities.
  In [95], the authors proposed the use of the homogeneous extended Jacobian matrix, which is derived for non-redundant parallel manipulators. The proposed method can be used for parallel manipulators with coupled rotations and translations by considering a set of linear independent axes at the points of a tetrahedron, which represents the permitted and restricted motions of the moving platform.

3.4. Difficulties related to the control of PMs

PMs have gained increased popularity in the last few decades. This interest has been stimulated by the significant advantages of PMs compared to their traditional serial counterparts, especially for some specific industrial tasks requiring high accelerations and accuracy. However, to fully exploit their potential and to get the most of their capabilities, a long path is still to be traversed. In addition to mechanical design, calibration, and optimisation of the structure, efficient control design plays an important role in improving the overall performance of PMs. However, PMs are known for their highly nonlinear dynamics, which increase considerably when operating at high accelerations, leading to mechanical vibrations. Moreover, uncertainties are abundant in such systems due to model simplifications, the wear of the components of the robot and the variations of the operating conditions. Furthermore, the coupled dynamics and actuation redundancy in some mechanisms give rise to complex and challenging control issues. Consequently, the

developed control schemes dedicated to PMs should take into account all these issues and challenges.
In the literature, several schemes have been proposed to resolve the problem of control of PMs. These control schemes include: PID control [96, 97]; improved NL-PID control [98]; computed-torque control [99]; augmented nonlinear PD control [100]; PD with computed feedforward [101]; dual-space adaptive control [6]; RISE-based adaptive control [102]; Extended DCAL [103]; Nonlinear dual-mode adaptive control [104]; adaptive terminal sliding mode control [105]; Extended L1 adaptive control [106] and others.

4. A new optimal dimensional design methodology for PMs

Based on previous works [44, 15], and given the various difficulties mentioned above, the methodology proposed to ensure an optimal design of fully parallel PMs can be described as follows:

- *Task Definition and Modelling;*
- *Choice of the mechanical architecture;*
- *Architecture modelling;*
- *Evaluation and analysis of geometric characteristics;*
- *Multi-objective optimisation of the chosen architecture;*
- *Choice of the final optimised solution;*
- *Definition of the tolerance values of the geometric parameters.*

These elements will be detailed in the sequel.

4.1. Task Definition and Modelling

In order to choose or design a manipulator to perform a given task according to a specification, we start by modeling this task, which represents the set of positions to be reached (a path to be covered, a set of crossing points, a surface or a workspace, etc.) under different constraints, namely [3]:

- The installation environment of the manipulator (accessibility limit, collisions, environmental specificities, etc.);
- Path following velocities;
- dynamics (payload, minimum acceleration or effort to be exerted by the PM);
- Environmental (specificity of the PM's environment, such as marine atmosphere or weightlessness).

4.2. Choice of the mechanical architecture

The choice of mechanical architecture depends on the nature of the tasks that the PM will have to perform. For instance, in pick-and-place operations, the task of the PM is to transport an object from one point to another at high speed (cf. illustration Figure 1). For these types of tasks, the PM can achieve accelerations of several tens of g [107]. In the field of High Speed Machining (cf. Figures 2 and 3), the tasks of the PM consist of machining surfaces (drilling, tapping, milling, etc.). A first phase of research will allow us to find the best topology (topology candidate).

4.3. Modelling of the chosen PM architecture

Modelling the mechanical architecture is an essential step in sizing the chosen architecture. It makes it possible to determine the different relationships between:

- The position and orientation of the mobile platform with respect to the articular joints;
- Joint and operational velocities;
- Forces/torques of the actuators required to produce the motion along a given trajectory (positions, joint velocities, and accelerations).

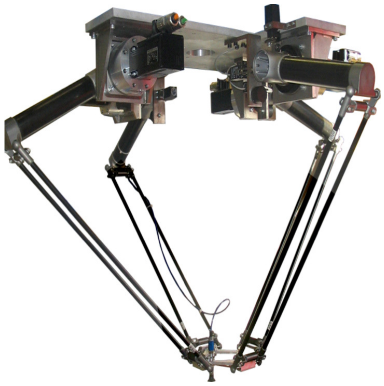

Figure 1: Illustration of a pick and place task with PAR44 Robot [107].

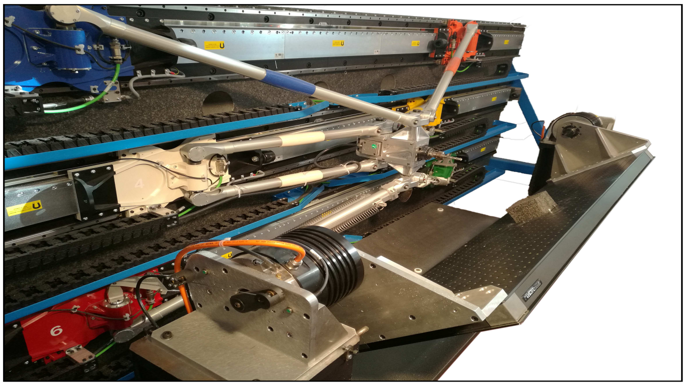

Figure 2: Illustration of the Arrow robot [106, 108].

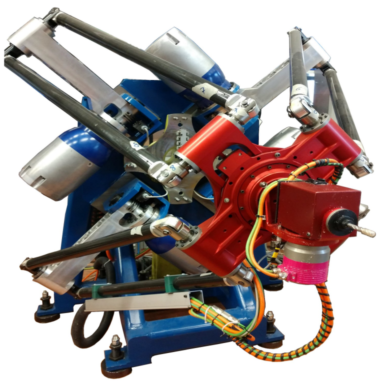

Figure 3: SPIDER4 Robot [109].

This phase also includes:

- Interference analysis: determination of actual workspace (limits of active and/or passive joints);
- Singularity analysis: it is of the utmost importance to carry out a singularity location analysis, to avoid when designing the PM;
- Determination of the payload;

- Determination of the driving efforts, according to: the external forces, the manipulator weight, the inertia forces, the friction forces, etc.

4.4. Evaluation of the performance criteria of the PM

Once the choice of the topology and its modelling is made, one proceeds to the study and evaluation of the performance criteria and the associated indicators. These performance criteria can be classified as follows:

- *The geometry of the PM*: the shape of the machine (topology), its workspace, size, positioning errors, collisions, etc.;
- *Kinematics*: dexterity, isotropy, precision, maximum velocities/accelerations of actuators and minimum guaranteed velocities/accelerations at the end point;
- *Statics*: The extreme forces generated by the actuators, the minimum forces guaranteed at the end-effector's reference point, the stiffness, the singular configurations, the energy efficiency, etc.;
- *Dynamics*: maximum accelerations at the actuators and minimum accelerations at the end-effector, with limited accelerations on the actuators (dynamic limits).

4.5. Multi-objective optimisation of the chosen architecture

The determination of the dimensions of the PM acting on its kinematics and dynamics represents the next stage. By this technique, dimensional synthesis is expressed in terms of multi-objective optimisation, taking into account several performance criteria simultaneously (such as workspace, kinematic performance, stiffness and dynamic performance, etc.).
Therefore, the associated optimisation problem can be mathematically formulated as follows:

$$\begin{array}{l}
\textbf{Find} \text{ a vector } P^* = [P_1^*, P_2^*, ..., P_n^*]^T \\
\text{That } \textbf{Minimizes/Maximizes } F(P) = [f_1(P), f_2(P), ..., f_k(P)]^T \\
\text{With } g_m(P) \leq 0 \ (m \text{ constraints of inequality}) \\
\text{and } h_l(P) = 0 \ (l \text{ constraints of equality}). \\
P^* \in \mathbb{R}^n \ : \text{Vector of the decision variables,} \\
F(P^*) \in \mathbb{R}^k \ : \text{Vector of the objective functions.}
\end{array}$$

In other words, what is the best vector P^* of the geometrical parameters of a PM that allows the largest dexterous workspace, the best kinematic performances, the best stiffness, the best precision, and the best dynamic performances at the same time?
The optimisation can be performed using metaheuristic methods (such as genetic algorithms, PSO, etc.) as a resolution approach. This choice is justified by the fact that these methods make it possible to determine an approximation of the entire Pareto front, even if the problem is not convex. Furthermore, they offer the possibility of avoiding local extrema.

4.6. Choice of the final optimised solution

The developed methodology allows for a set of optimal solutions. This may raise a new problem, which is how to choose the ultimate optimal solution (a single optimised vector of geometric parameters). Several solutions are possible at this stage:

- The intervention of the designer (user) to choose the optimised geometric vector;
- The introduction of a new classification criterion (the preference of the designer for one of the objectives);
- The introduction of decision aid methods (based on establishing an orderly relationship between the different objectives).

4.7. Tolerance synthesis of optimised geometric parameters

Since the performance criteria of PMs are highly sensitive to their geometric parameters, it is wise to ensure that the robot keeps these performance criteria.
The objective of this step is to integrate the variations of the optimised geometric parameters (dimensions) in order to predict the problems related to these variations and to minimise their effects on the performance criteria of the PM [110, 111].
It is, therefore, necessary to study the effects of small changes in the design solution, in order to define the tolerance intervals of optimised dimensions. In addition, the calculation of the tolerances of parts in the manufacturing and assembly phases of robots is essential in their design because it directly affects the quality of manufacturing cost of the PM.

4.8. Summary

The proposed design methodology of PMs can be summarized in the flowchart of Figure 4:

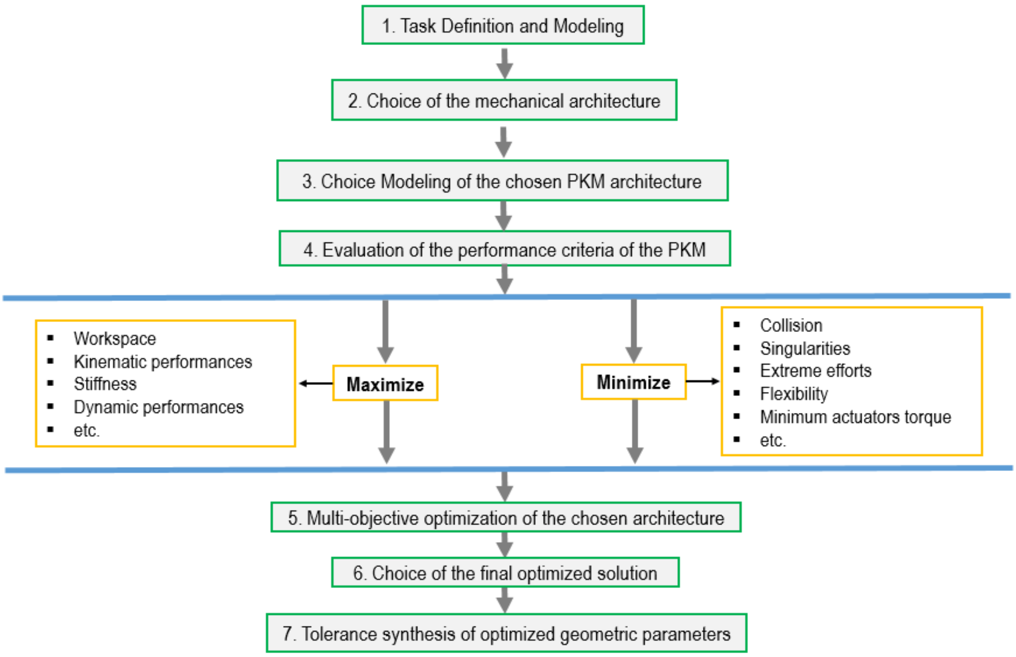

Figure 4: Illustrative flowchart describing the proposed design methodology.

5. Application of the proposed methodology: Case study

In this example, we apply the developed methodology to the design of a 5R parallel manipulator with 2-DOFs (cf. illustration of Figure 5).

- **Stage 1 (Task Definition and Modeling)**:
  The task is carried out in the manufacturing industry of micro-electronic products and the assembly of electronic components. This task requires extreme accuracy to ensure better product quality and high speed to ensure productivity. The task can be represented by the intervention in a workspace of $40cm \times 40cm$.

- **Stage 2 (Choice of the mechanical architecture)**:
  The type used for this type of task was the SCARA serial architecture. Currently, PMs have shown their performance in terms of precision and speed. For this purpose, for the present task, we propose to choose the manipulator 5R (SCARA with double arm). It is with a parallel architecture with 2 DOFs (cf. illustration of

Figure 5).

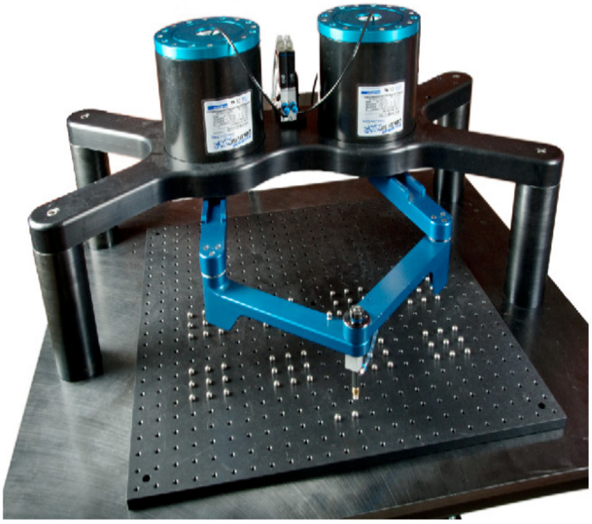

Figure 5: View of the DexTAR parallel manipulator [112].

- **Stage 3 (Architecture modeling):**
  The 5R planar PM has been the subject of several studies in the literature [112–121]. It has a single closed

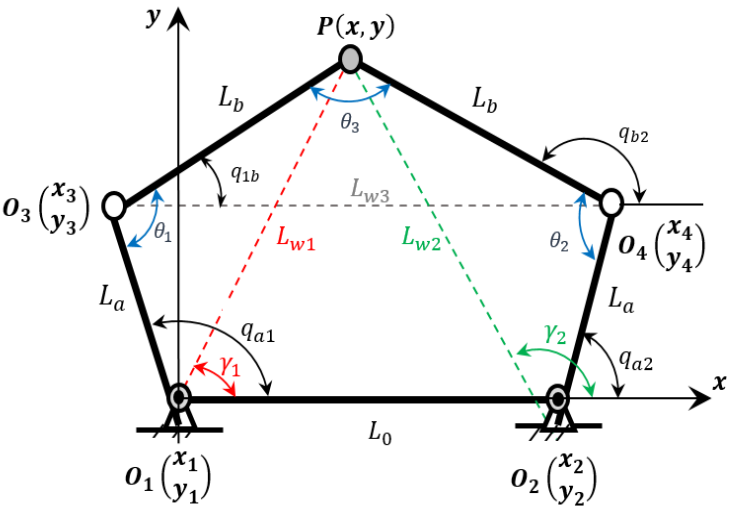

Figure 6: Parametrization of the parallel manipulator 5R.

kinematic chain and its effector is placed at $P(x, y)$. The motorized joints q_{a1} and q_{a2} are represented by pivots O_1 and O_2 and the passive joints q_{b1} and q_{b2} are represented by pivots O_3, O_4 and P. For the purpose of simplification of the realization of the architecture, it has been considered that the two branches O_1O_3P and O_2O_4P are identical. Therefore, the dimensions of this manipulator are: $O_1O_2 = L_0$, $O_1O_3 = O_2O_4 = L_a$, $O_3P = O_4P = L_b$. The vector of parameters to be optimized is: $P^* = [L_0^*, L_a^*, L_b^*]^T$.

The coordinates of the points $O_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix}$, for $i = 1, \cdots, 4$ are as follows:

$O_1 = \begin{pmatrix} x_1 = 0 \\ y_1 = 0 \end{pmatrix}, O_2 = \begin{pmatrix} x_2 = L_0 \\ y_2 = 0 \end{pmatrix}, O_3 = \begin{pmatrix} x_3 = L_a cosq_{a1} \\ y_3 = L_a sinq_{a1} \end{pmatrix}, O_4 = \begin{pmatrix} x_4 = L_0 + L_a cosq_{a2} \\ y_4 = L_a sinq_{a2} \end{pmatrix}$

From Figure 6, we can deduce:

$$L_b^2 = (x - x_3)^2 + (y - y_3)^2 \tag{17}$$

$$L_b^2 = (x - x_4)^2 + (y - y_4)^2 \tag{18}$$

By replacing x_i and y_i in the above expressions, we obtain:

$$L_b^2 = (x - L_a cosq_{a1})^2 + (y - L_a sinq_{a1})^2 \tag{19}$$

$$L_b^2 = (x - L_0 - L_a cosq_{a2})^2 + (y - L_a sinq_{a2})^2 \tag{20}$$

Combining (19)-(20), gives:

$$x = ay + b \tag{21}$$

with:

$a = \frac{L_a(sinq_{a1} - sinq_{a2})}{L_0 + L_a cosq_{a2} - L_a cosq_{a1}}$; $b = \frac{L_0^2 + 2L_0 L_a cosq_{a2}}{2(L_0 + L_a cosq_{a2} - L_a cosq_{a1})}$

If (21) is replaced in (20), it leads to :

$$cy^2 + dy + e = 0 \tag{22}$$

Hence, the solution is given by:

$$y = \frac{-d \pm \sqrt{d^2 - 4ce}}{2c} \tag{23}$$

with:
$c = a^2 + 1$
$d = 2(ab - aL_a cosq_{a1} - L_a sinq_{a1})$
$e = L_a^2 - L_b^2 - 2bL_a cosq_{a1}$

The time derivative of equations (19) and (20), leads to the Jacobian matrices:

$$J_x = \begin{pmatrix} x - L_a cosq_{a1} & y - L_a sinq_{a1} \\ x - L_0 - L_a cosq_{a2} & y - L_a sinq_{a2} \end{pmatrix} \tag{24}$$

$$J_q = \begin{pmatrix} (ycosq_{a1} - xsinq_{a1})L_a & 0 \\ 0 & (ycosq_{a2} - (x - L_0)sinq_{a2})L_a \end{pmatrix} \tag{25}$$

Hence,

$$J^{-1} = J_q^{-1} J_x \tag{26}$$

The analysis of the inverse Jacobian matrix allows us to avoid singular configurations. For more details, the reader can refer to [122].

- The working mode:
In [123], the authors introduced the notion of working modes and applied it for analyzing the different working modes of the 5R manipulators in the cartesian workspace and in the joint space, which include orientation, translation, and combined motions, and discussed the various aspects that need to be considered when designing and controlling them. They presented a detailed analysis of these aspects, including workspace, dexterity, singularity, and trajectory planning.
In this case, in equation (23), we considered the sign (+) because the robot works in the positive half-plane ($y = 0$).

- **Stage 4 (Evaluation and analysis of geometric characteristics):**

  - **Workspace**
    The workspace is the intersection of the two areas formed by the rays $L(w_1)$ and $L(w_2)$ and the centers O_1 and O_2, that is:

$$S_w =\mid w_1 - w_2 \mid= \frac{1}{2}(\mid \int_{\gamma_{1_{min}}}^{\gamma_{1_{max}}} (L_{w_1})^2 d\gamma_1 - \int_{\gamma_{2_{min}}}^{\gamma_{2_{max}}} (L_{w_2})^2 d\gamma_2 \mid) \tag{27}$$

$$S_w = \frac{1}{2}[L_{w_1}^2(\gamma_{1_{max}} - \gamma_{1_{min}}) - L_{w_2}^2(\gamma_{2_{max}} - \gamma_{2_{min}})] \tag{28}$$

with:
$q_{1_{min}} \leq \gamma_1 \leq q_{1_{max}}$
$q_{2_{min}} \leq \gamma_2 \leq q_{2_{max}}$
From Figure 6, the following relationship can be deduced:
$L_{w_1}^2 = L_a^2 + L_b^2 - 2L_aL_bcos\theta_1$
$L_{w_2}^2 = L_a^2 + L_b^2 - 2L_aL_bcos\theta_2$
$0 \leq \theta_1 \leq \pi$
$0 \leq \theta_2 \leq \pi$
$0 \leq \theta_3 \leq \pi$
Where:
$\sqrt{L_a^2 + L_b^2 - 2L_aL_b} < L_{w_1} < \sqrt{L_a^2 + L_b^2 + 2L_aL_b}$
$\sqrt{L_a^2 + L_b^2 - 2L_aL_b} < L_{w_2} < \sqrt{L_a^2 + L_b^2 + 2L_aL_b}$
$0 < L_{w_3} < \sqrt{2}L_b$
PMs are generally characterized by a relatively small workspace, compared to their serial counterparts. Therefore, our goal would be to maximize this workspace during this phase. Accordingly, the objective function displayed is:

$$S_w \rightarrow max \quad \Leftrightarrow \quad -S_w \rightarrow min \tag{29}$$

  - **Kinematic performance**
    The global kinematic isotropy index η_j (see equation (5)), is used to evaluate the kinematic performance of the 5R PM. Therefore, the objective function is given by the following expression:

$$\eta_j \rightarrow min \tag{30}$$

- **Stage 5 (Multi-objective optimization of the chosen architecture):**

  - ***Problem Formulation***

Find a vector $P^* = [L_0^*, L_a^*, L_a^*]^T$

that: **Minimize** $F(P) = \begin{bmatrix} -S_w(P) \\ \eta_j(P) \end{bmatrix}$

Subject to:

$0 \leq q_{a1} \leq 2\pi$
$-\pi \leq q_{a2} \leq \pi$
$\frac{\pi}{6} \leq \theta_1 \leq \frac{5\pi}{6}$
$\frac{\pi}{6} \leq \theta_2 \leq \frac{5\pi}{6}$
$0.25m \leq L_0 \leq 0.5m$
$0.1m \leq L_a \leq 0.2m$
$0.275m \leq L_b \leq 0.4m$

– ***Results and discussion***

To resolve the above optimization problem, evolutionary algorithms are the alternative since they enable us to compute the full Pareto front even when the problem is non-convex. Additionally, they provide a way around local extrema as a hurdle. Particularly we propose to use the SPEA-II algorithm by Zigler et al. [124] because it has shown a certain effectiveness in a variety of application areas. The software Matlab is used for the implementation of the SPEA-II algorithm. Beside, it is worth to note that there is a conflict between the kinematics performance n_j and the workspace S_w as shown in Figure 7.

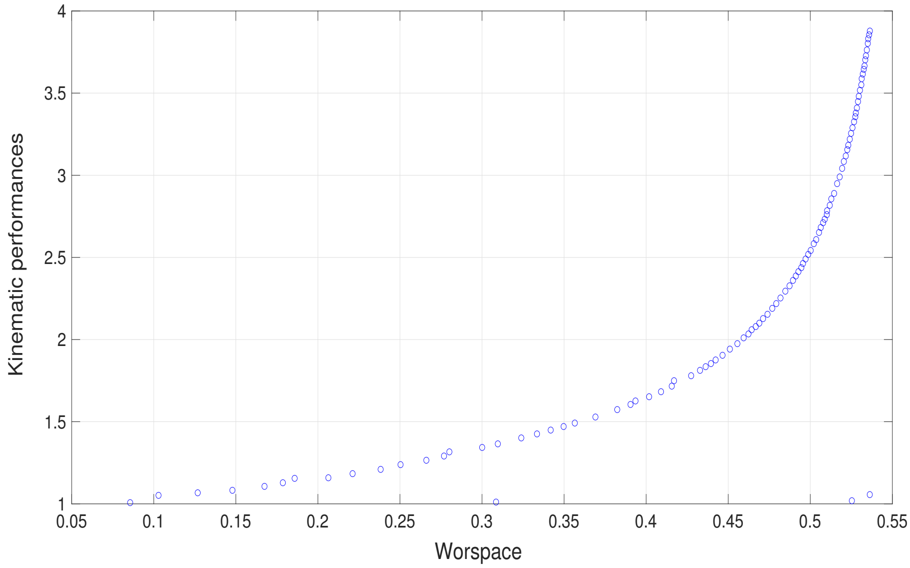

Figure 7: Front of Pareto between the workspace S_w and the kinematic performances n_j.

- **Stage 6 (Choice of the final optimized solution):**

  The multi-objective optimization method allows us to obtain a set of optimal solutions, none of which can be distinguished as the best without introducing a new classification criterion (i.e. the designer's preference for one of the objectives). To this end, TOPSIS Algorithm [125–127] can be used. Consequently, the chosen vector of objectives is the following: $\begin{bmatrix} S_w(P) = 0,2493847 \ m^2 \\ \eta(P) = 3,1141387 \end{bmatrix}$

  which corresponds to : $\begin{bmatrix} L_0^* = 0,4646297 \ m \\ L_a^* = 0,1994198 \ m \\ L_b^* = 0,3994085 \ m \end{bmatrix}$

- **Stage 7 (Definition of the tolerance values of the geometric parameters):**

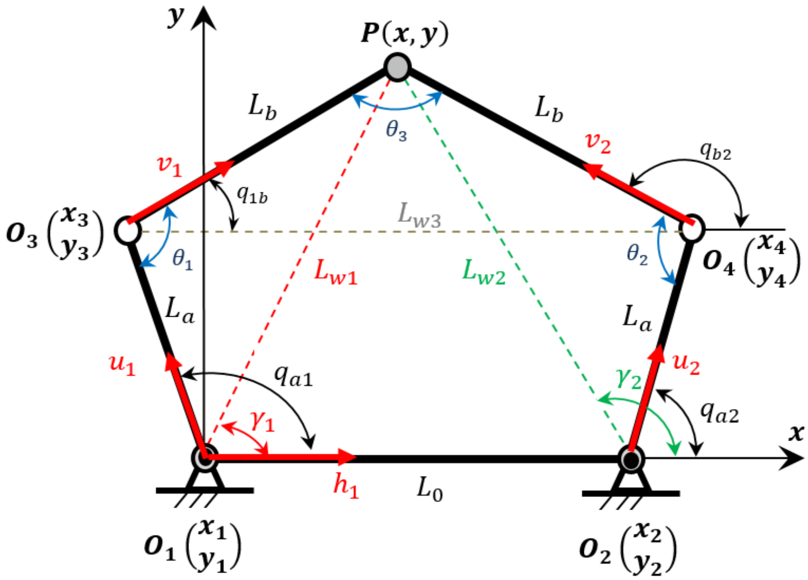

Figure 8: Illustration of the angles delimiting the manipulator's workspace.

Based on the illustration of Figure 8, we can deduce:

$$\overrightarrow{O_1P} = L_0\overrightarrow{h_i} + La_i\overrightarrow{u_i} + Lb_i\overrightarrow{v_i}, \qquad i = 1, \cdots 2 \tag{31}$$

Where:
$\overrightarrow{h_i}$ is the unit vector $\frac{\overrightarrow{L_0}}{\|\overrightarrow{L_0}\|_2}$
$\overrightarrow{u_i}$ is the unit vector $\frac{\overrightarrow{La_i}}{\|\overrightarrow{La_i}\|}$,
$\overrightarrow{v_i}$ is the unit vector $\frac{\overrightarrow{Lb_i}}{\|\overrightarrow{Lb_i}\|}$ That can be expressed as follows:

$$\begin{cases} \overrightarrow{h_1} = \vec{0}, \overrightarrow{h_2} = \vec{0} \\ \overrightarrow{u_1} = cos(q_1)\vec{x} + sin(q_1)\vec{y}, \overrightarrow{u_2} = cos(q_2)\vec{x} + sin(q_2)\vec{y} \\ \overrightarrow{v_1} = cos(q_{b1})\vec{x} + sin(q_{b1})\vec{y}, \overrightarrow{v_2} = cos(q_{b2})\vec{x} + sin(q_{b2})\vec{y} \end{cases} \tag{32}$$

If we consider only the dimensional variations, we obtain after differentiation of equation (26) the following relationship:

$$\delta\overrightarrow{O_1P} = \delta L_0\overrightarrow{h_1} + \delta La_i\overrightarrow{u_i} + \delta Lb_i\overrightarrow{v_i} \tag{33}$$

Where:
$\delta\overrightarrow{O_1P}$ denotes the variation of the end effector position, its components are: $[\delta x\ \delta y]^T$
δL_0 denotes the variation of the lenght L_0 (nominal lenght is $L_0 = 0.464629m$)
δLa_i denotes the variation of the lenght La_i (nominal lenght is $La_i = La = 0.199419m$)
δLb_i denotes the variation of the lenght Lb_i (nominal lenght is $Lb_i = Lb = 0.399408m$)
The relationship between the variation of the position error of the end effector P, and the dimensional variations δL_0, δLa_i, and δLb_i, can be avec expressed by the following equation:

$$\delta P = J_y\delta y \tag{34}$$

Table 1
Summary of the obtained optimized tolerances μm.

ΔLa_1	ΔLb_1	ΔL_0	ΔLa_2	ΔLb_2
3.3359	0.6898	0.9689	6.6833	0.6102

Such as:

$$J_y = \begin{pmatrix} 1 & cos(q_1) & cos(q_{b1}) & cos(q_2) & cos(q_{b2}) \\ 0 & sin(q_1) & sin(q_{b1}) & sin(q_2) & sin(q_{b2}) \end{pmatrix} \tag{35}$$

and

$$\delta y = \begin{bmatrix} \delta L_0 & \delta La_1 & \delta Lb_1 & \delta La_2 & \delta Lb_2 \end{bmatrix}^T_{1\times 5} \tag{36}$$

The sensitivity matrix $S = J_y^T J_y$ [110] is a semi-positive definite matrix, hence $rank(S) = 2 < n$ ($n = 5, \lambda_1 = \lambda_2 = \lambda_3 = 0$). Equation (34) represents a family of hyper-cylindroids, each cylindroid has two infinite principal axes.
From the matrix S, the eigenvalues of the most constraining cylindroid can be calculated. They are given by: $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 = 0.604, \lambda_5 = 4.3959$.
As $\delta f = J_y \delta y$, one can deduce that $Y_r^2 = (0.01)^2 = 0.0001(\|\delta f\|_2^2 = (0.01)^2)$, which is an imposed value.
Let us now take the modification coefficient $K = 0.03$ [128], [129]. The eigenvalues of the feasible space can then be given by $\hat{\lambda}_1 = \hat{\lambda}_2 = \hat{\lambda}_3 = 0.1808$, $\hat{\lambda}_4 = 0.604$, $\hat{\lambda}_8 = 4.3959$. With the eigenvalues and eigenvectors P_i previously obtained, the characteristic matrix $\hat{S} = P\hat{D}P^T$ [110] can then be constructed. The most constraining sensitivity ellipsoid is denoted by P_{sy}. The manipulator posture equivalent to this position is shown at point P10 in Figure 9. To calculate the dimensional tolerances $L_{0_{opt}}$, La_{iopt}, and Lb_{iopt}, of lengths L_0, La_i, and Lb_i, respectively, the following optimization problem needs to be resolved:

$$\begin{cases} \max\limits_{u} |u_1 u_2 u_3 u_4 u_5| \\ Suchas : U(u_1, u_2, u_3, u_4, u_5) \in \zeta_{crit} \\ u_j.Sing(V_j) \geq 0, j = 1, \cdots, 5 \\ |u_j| \geq 0,5\mu m, j = 1, \cdots, 5 \end{cases} \tag{37}$$

The constraint $|u_i| \geq 0,5\ \mu m$ is the tolerated dimensional tolerance of the variables δL_0, δLa_i, and δLb_i. The solution of this optimization problem is calculated using the *fmincon* function of Matlab software. The obtained results are summarized in Table 1 ($-L_0 \leq \delta L_0 \leq L_0$, $-\Delta La_i \leq \delta La_i \leq \Delta La_i$, $-\Delta Lb_i \leq \delta Lb_i \leq \Delta Lb_i$ $i = 1, 2$).

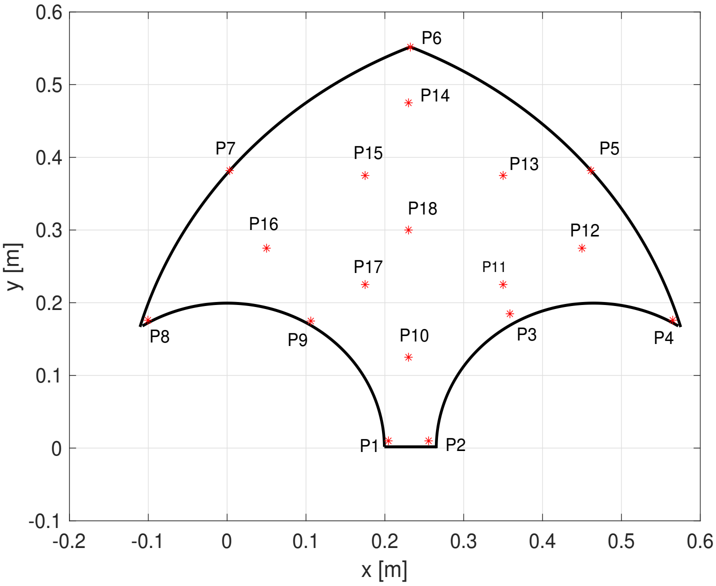

Figure 9: Illustration of the 5R manipulator Workspace.

6. Conclusions and future research directions

In this article, the main approaches used in the dimensional synthesis of PMs are presented. The performance criteria and indices such as workspace, kinematic performance, stiffness, and dynamic performance, were exposed. The interest of the homogenisation in the Jacobian matrix, as well as the analysis of the singularities, is shown. Then, a review of these methods was presented, in addition to the difficulties resulting from open problems encountered during the design stage. Finally, a new methodology for the optimal dimensional design of PMs was proposed. It is noteworthy to mention that this design methodology is based on a multi-objective function. In order to test the efficiency of this methodology, we have applied it to a 5R parallel manipulator with 2-DOFs. It allows for a good approximation of the compromise between the workspace and the kinematic performance, which are antagonistic. The TOPSIS algorithm was then used to select the final optimised solution. At the end, the tolerance values of the selected geometric parameters have been established. Furthermore, it is important to emphasise that this methodology is applicable to full PMs, regardless of their number of degrees of freedom. However, many interesting questions remain open, such as how to choose a PM for a given task (meeting the requirements of technical specifications)? In addition, the difficulties encountered (such as those related to the kinematic modelling, the determination of the workspace, the vibrations of the manipulator structure and those related to the control of these PMs) must also be adressed.

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Appendix: SPEA-II Algorithm

The principle of SPEA-II is based on the following algorithm [124]:

- The inputs of the algorithm are: N: population size, $\overline{N}$: archive size, T: maximum number of generations
- The output of the algorithm is: A: the non-dominated set
- The five steps of the algorithm are as follows:

Step 1: Initialisation, generate an initial population P_0 and create the empty archive $\overline{P_0}$ set $t = 0$.
Step 2: Fitness assignment: Calculate fitness values of individuals in P_t and $\overline{P_t}$.
Step 3: Environmental selection: Copy all non-dominated individuals in P_t and $\overline{P_t}$ to $\overline{P}_{t+1}$. If size of $\overline{P}_{t+1}$ exceeds $\overline{N}$ then reduce $\overline{P}_{t+1}$ by means of the truncation operator, otherwise if size of $\overline{P}_{t+1}$ is less than $\overline{N}$ then fill $\overline{P}_{t+1}$ with dominated individuals in P_t and $\overline{P_t}$.
Step 4: Termination: If $t \geq T$ or another stopping criterion is satisfied then set A to the set of decision vectors represented by the non-dominated individuals in $\overline{P}_{t+1}$. Stop.
Step 5: Mating selection: Perform binary tournament selection with replacement on $\overline{P}_{t+1}$ in order to fill the mating pool.