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# Designs of BCD Adder Based on Excess-3 Code in Quantum-dot Cellular Automata 

Aibin Yan, Runqi Liu, Zhixing Li, Jie Cui, Tianming Ni, Patrick Girard, Fellow, IEEE and Xiaoqing Wen, Fellow, IEEE


#### Abstract

Quantum-dot cellular automata (QCA) is a novel nano-electronic technology. QCA has attracted wide attention due to its extremely small feature sizes at the molecular or even atomic scale and ultra-low power consumption, making it a promising candidate to replace the complementary metal oxide semiconductor (CMOS) technology. Binary-Coded Decimal (BCD) adders are widely used in industrial computing. In this paper, we propose two types of excess-3 code (XS-3) based BCD adders (XS-3DAs). We use ripple-carry adders (RCA) and parallel binary adders (PBA) to construct XS-3DAs in QCA Designer tool, respectively. The PBA-based XS-3DA is constructed with a new correction logic. 4-bit, 8 -bit, and 16-bit XS-3DAs are constructed based on the two proposed XS-3DAs, respectively. Comparisons show that, with the increase of design scaling, the delay and area-delay product (ADP) of the PBAbased XS-3DAs can be significantly reduced in comparison with that of the RCA-based XS-3DAs. Compared with the 16-digit RCA-based XS-3DA, the cell count, area, delay and ADP of the proposed 16-digit PBA-based XS-3DA are reduced by $\mathbf{3 7 . 8 8 \%}$, $\mathbf{2 5 . 9 9 \%}$, $\mathbf{3 7 . 6 8 \%}$ and $53.88 \%$, respectively.


Index Terms-Quantum-dot cellular automata, Excess-3 code, BCD adder

## I. Introduction

The processing of all data in computers is essentially the processing of binary numbers. Only binary numbers can be directly recognized and processed by computers, and people are more accustomed to using decimal numbers in practice. Therefore, there is a strong need for computers to directly process data expressed in decimal form.

In the near future, the CMOS technology will encounter bottlenecks due to its short channel effect and the high lithography cost. Nanotechnologies, especially QCA, have

[^0]become the promising alternatives, which provide new choices for the design and manufacture of integrated circuits. The QCA technology is a widely studied nanotechnology. Its high device density, fast switching speeds and ultra-low energy consumption indicate enormous research potentials and broad application prospects.

The concept and advantages of XS-3 is briefly introduced here. The XS- 3 is formed by the addition of 8421 BCD codes and (0011)2, which is a self-complementary BCD code. This code has a biased representation. It is easy to be 9's complemented by inverting all bits, which facilitates the calculation. For example, the XS-3 of 2 are 0101 and the binary number after reversing each digit is 1010 that is exactly the XS-3 of 7. Moreover, when the sum of two XS-3s is greater than 9 , the carry bit of a 4-bit binary adder can be set high. So far, researchers have proposed some QCA-based BCD converters, i.e., 8421 BCD code to the XS-3 converters and XS-3 to 8421 BCD code converters [1-2]. The advantages of XS-3 motivate us to use it to design BCD adders.

There are many previous works on BCD adders. In 2008, Taghizadeh et al firstly proposed a modular design of a BCD adder in QCA [3]. In 2017, Abedi et al proposed a QCA decimal full adder that is mostly composed of fully utilized majority gates (i.e., with no constant inputs), and rarely includes partially utilized majority gates [4]. A new correction logic formulation was proposed to reduce the delay of single and multi-digit BCD adders [5]. In 2018, Zhang et al proposed a carry look ahead structure for calculating all intermediate output carries and used this method to design multi-digit decimal adders [6]. In 2018, Li et al proposed a new structure for BCD adders. It achieved a more compact layout by making some equivalent replacement and introducing XOR operations [7]. In 2021, Chu et al proposed a BCD adder based on 3-input XOR gates and 3-input majority gates [8]. The introduction of 3-input XOR gates is conducive for achieving compact logic representations. All of the above BCD adders are designed based on 8421 BCD code.

To the best of our knowledge, there is no QCA-based XS3DAs in the literature. In this paper, we propose RCA-based and PBA-based XS-3DAs. The PBA-based XS-3DA is constructed using a new correction logic. The RCA-based and PBA-based XS-3DAs are implemented in the QCA Designer tool [9]. We analyze the characteristics of the proposed two types of XS-3DAs. Comparisons show that, with the increase of design scaling, the delay and ADP of the PBA-based XS3DAs can be significantly reduced compared with that of RCA-based XS-3DAs.

The rest of the paper is organized as follows: Section II introduces the preliminaries of the QCA technology. Section III presents RCA-based and PBA-based XS-3DAs. Section IV details the simulation results and comparisons. Section V concludes this paper.

## II. Preliminaries of QCA

QCA cells are the basic unit in QCA. A common QCA cell is composed of two free electrons and four quantum dots. Due to the effect of electrostatic force, the free electrons in a cell can eventually get into two stable states, which represent two binary states. The unique clock mechanism in QCA ensures the stable transmission of signals. Figure 1 shows the cells and clock mechanism of QCA. A QCA clock has four clock phases: switch, hold, release, and relax. In the switch phase, the QCA cells enter the polarization state. In the hold phase, the QCA cells reaches the maximum polarity and remain unchanged. In the release phase, the polarity of the QCA cells begins to decrease. In the relax phase, the QCA cells completely lose polarization and reach a non-polarized state.


Fig. 1. QCA cells and QCA clock. (a) QCA cells and (b) QCA clock.
The basic logic devices in QCA are 3-input majority gates and inverters. Figure 2 shows some devices in QCA. The inverter can reverse the input signal, and the 3 -input majority gate can output the signal that follows the majority rule as shown in Fig. 2 (a)-(b). The XOR function can be realized by a QCA-based gate, which is different from the construction of the XOR gate in the CMOS circuit. Figure 2 (c) shows a 3input XOR gate.


Fig. 2. QCA basic gates. (a) inverter, (b) 3-input majority gate and (c) 3-input XOR gate.

## III. PROPOSED RCA-BASED AND PBA-BASED XS-3DAs

## A. RCA-based XS-3DAS

The XS-3 is formed by the addition of 8421 BCD codes and $(0011)_{2}$. When two decimal numbers represented by XS3 s are added, the carry can be generated correctly, but the sum must be corrected. The correction method is as follows. If the carry is " 1 ", the sum needs to be added with $(0011)_{2}$; if the
carry is " 0 ", the sum needs to be added with (1101) 2 . The two correction signals $(0011)_{2}$ and $(1101)_{2}$ can be generated by the correction logic circuit.

Figure 3 shows the block diagram of the 1-digit RCA-based XS-3DA. It can be seen that, the 1-digit RCA-based XS-3DA includes 4-bit binary adder ( $\mathrm{ADD}_{1}$ ), correction logic (CL) and 4-bit binary adder $\left(\mathrm{ADD}_{2}\right)$. The signals $\mathrm{dA}_{3: 0}$ and $\mathrm{dB}_{3: 0}$ are two decimal input numbers represented by excess-3 codes. The signals $\mathrm{dCout}_{\text {out }}$ and $\mathrm{dS}_{3: 0}$ are the carry and sum of the 1-digit XS3DA. The $\mathrm{ADD}_{1}$ adds the three signals of $\mathrm{dA}_{3: 0}, \mathrm{~dB}_{3: 0}$ and $\mathrm{dC}_{\mathrm{in}}$ to obtain binary $\mathrm{bS}_{3: 0}$ and carry $\mathrm{bC}_{\text {out. }}$. At this time, the carry $\mathrm{bC}_{\text {out }}$ is correct and does not need to be corrected. The function of the CL circuit is to generate the correction signal. It is composed of an inverter, which can generate $\mathrm{cL}_{3: 0}$. When $\mathrm{bC}_{\text {out }}$ is $1, \mathrm{cL}_{3: 0}$ is $(0011)_{2}$; when $\mathrm{bC}_{\text {out }}$ is $0, \mathrm{cL}_{3: 0}$ is $(1101)_{2} . \mathrm{ADD}_{2}$ adds $\mathrm{cL}_{3: 0}$ and $\mathrm{bS}_{3: 0}$ to get decimal $\mathrm{dS}_{3: 0}$.


Fig. 3. Block diagram of 1-digit RCA-based XS-3DA.
The delay of the outputs ( $\mathrm{C}_{\text {out }}$ and $\mathrm{dS}_{3: 0}$ ) of the 1-digit RCAbased XS-3DA can be expressed by formulas (1)-(2).

$$
\begin{gather*}
\mathrm{D}(1)_{\mathrm{C}}=\mathrm{D}_{\mathrm{ADD} 1}  \tag{1}\\
\mathrm{D}(1)_{\mathrm{S}}=\mathrm{D}_{\mathrm{ADD} 1}+\mathrm{D}_{\mathrm{CL}}+\mathrm{D}_{\mathrm{ADD} 2} \tag{2}
\end{gather*}
$$

$\mathrm{D}_{\mathrm{ADD} 1}, \mathrm{D}_{\mathrm{CL}}$ and $\mathrm{D}_{\mathrm{ADD} 2}$ represent the delay of $\mathrm{ADD}_{1}, \mathrm{CL}$ and $\mathrm{ADD}_{2}$, respectively.

The delay of the outputs ( $\mathrm{C}_{\text {out }}$ and $\mathrm{dS}_{3: 0}$ ) of the n-digit RCAbased XS-3DA can be expressed by formulas (3)-(4).

$$
\begin{gather*}
\mathrm{D}(\mathrm{n})_{\mathrm{C}}=\mathrm{nD}_{\mathrm{ADD} 1}  \tag{3}\\
\mathrm{D}(\mathrm{n})_{\mathrm{S}}=\mathrm{nD}_{\mathrm{ADD} 1}+\mathrm{D}_{\mathrm{CL}}+\mathrm{D}_{\mathrm{ADD} 2} \tag{4}
\end{gather*}
$$



Fig. 4. Layout of the proposed 1-digit RCA-based XS-3DA.
Previous studies have proved that the introduction of XOR logic to construct full adders can effectively reduce overhead of QCA circuits [8]. We use a 3-input XOR gate and a 3-input majority gate to propose a full adder (see Pro-FA in Fig. 4 and Table II) to reduce circuit overhead. Figure 4 shows the layout of the proposed 1-digit RCA-based XS-3DA in the QCA

Designer tool. The cells marked with " $\bigcirc$ " are on the second layer, and the cells marked with " $X$ " are on the third layer. The $\mathrm{dA}_{3: 0}$ and $\mathrm{dB}_{3: 0}$ are two groups of input signals. The $\mathrm{dC}_{\mathrm{in}}$ represents the input carry signal, $\mathrm{dC}_{\text {out }}$ represents the output carry signal, and $\mathrm{dS}_{3: 0}$ represents the sum signal. The design consists of 539 cells, with an area of $0.60 \mathrm{um}^{2}$, a delay of 2.25 clocks for signal $\mathrm{dS}_{3: 0}$, and a delay of 1.25 clocks for the signal $\mathrm{dC}_{\text {out }}$. It is a 3-layer design. $\mathrm{ADD}_{1}$ and $\mathrm{ADD}_{2}$ of the proposed 1-digit RCA-based XS-3DAs are implemented using the proposed FAs. Because the signal $\mathrm{cL}_{0}$ is always " 1 ", the rightmost FA of $\mathrm{ADD}_{2}$ can be replaced by an inverter. N-digit RCA-based XS-3DAs can be constructed using the proposed RCA-based XS-3DAs.

## B. PBA-based XS-3DAS

Figure 5 shows a block diagram of BCD adders [6]. Ajitha et al and Zhang et al have applied the block diagram to design BCD adders based on 8421 BCD codes, respectively [5-6]. We apply this method for the construction of XS-3DAs.

The following is the analysis of the proposed PBA-based XS-3DAs. The signals $\mathrm{dA}_{3: 0}$ and $\mathrm{dB}_{3: 0}$ are two decimal input numbers represented by excess-3 codes. It can be seen from Fig. 5 that, after adjusting the input of signal $\mathrm{dC}_{\mathrm{in}}$, when the signal $\mathrm{bS}_{3: 0}$ is greater than or equal to " 10 ", the carry signal $\mathrm{bC}_{\text {out }}$ is correct no matter the value of $\mathrm{dC}_{\mathrm{in}}$; when $\mathrm{bS}_{3: 0}$ is less than " 9 ", the carry signal $\mathrm{bC}_{\text {out }}$ is correct no matter the value of $\mathrm{dC}_{\mathrm{in}}$; when $\mathrm{bS}_{3: 0}$ is equal to " 9 " and $\mathrm{dC}_{\mathrm{in}}$ is equal to " 0 ", the carry signal $\mathrm{bC}_{\text {out }}$ is correct; when $\mathrm{bS}_{3: 0}$ is equal to " 9 " and $\mathrm{dC}_{\text {in }}$ is equal to " 1 ", the signal $\mathrm{bC}_{\text {out }}$ is " 0 ", which is wrong and needs to be corrected. The output carry $\mathrm{dC}_{\text {out }}$ of the XS-3DAs can be rewritten as formula (5).


Fig. 5. Block diagram of 1-digit BCD adder [8].

$$
\begin{equation*}
\mathrm{dC}_{\text {out }}=\mathrm{bC}_{\text {out }}+\left(\mathrm{bS}_{3: 0}==9\right) \mathrm{dC}_{\mathrm{in}} \tag{5}
\end{equation*}
$$

The excess-3 code of "9" is (1111) xs-3. Formula (5) can be rewritten as formula (6).

$$
\begin{align*}
& \mathrm{dC}_{\text {out }}=\mathrm{bC}_{\text {out }}+\left(\mathrm{bS}_{3: 0}==(1111)_{\mathrm{XS}-3}\right) \mathrm{dC}_{\text {in }} \\
& \mathrm{dC}_{\text {out }}=\mathrm{bC}_{\text {out }}+\mathrm{bS}_{3} \cdot \mathrm{bS}_{2} \cdot \mathrm{bS}_{1} \cdot \mathrm{bS}_{0} \cdot \mathrm{dC}_{\text {in }} \\
& \mathrm{dC}_{\text {out }}=\mathrm{bC}_{\text {out }}+\left(\mathrm{bS}_{3} \cdot \mathrm{bS}_{2}\right) \cdot\left(\mathrm{bS}_{1} \cdot \mathrm{bS}_{0}\right) \cdot \mathrm{dC}_{\text {in }} \tag{6}
\end{align*}
$$

Figure 6 shows the proposed CL circuit of PBA-based XS3DAs. The CL circuit of the proposed PBA-based XS-3DAs can process signals $\mathrm{bS}_{3: 0}, \mathrm{bC}_{\text {out }}$ and $\mathrm{dC}_{\text {in }}$ to obtain signals $\mathrm{dC}_{\text {out }}$. Figure 7 shows the schematic diagram of the proposed PBAbased XS-3DAs. Compared with the CL circuit of the RCAbased XS-3DAs, although the proposed CL circuit of the PBA-based XS-3DAs has five more gates, the signal $\mathrm{dC}_{\mathrm{in}}$ can flow directly into the CL circuit without flowing through $\mathrm{ADD}_{1}$ and each $\mathrm{ADD}_{1}$ of $n$-digit XS-3DA can be operated in parallel to reduce delay. Theoretically, when $\mathrm{dC}_{\text {in }}$ is inputted, the carry signal $\mathrm{dC}_{\text {out }}$ can be output after a delay of 0.5 clocks
in QCA circuit, which can significantly reduce the delay of the n-digit XS-3DAs.


Fig. 6. The proposed CL circuit of PBA-based XS-3DAs.


Fig. 7. Circuit diagram of the proposed 1-digit PBA-based XS-3DA.
The delay of the outputs ( $\mathrm{C}_{\text {out }}$ and $\mathrm{dS}_{3: 0}$ ) of the proposed 1digit PBA-based XS-3DA can be expressed by formulas (7)(8). For brevity, the CL circuit is divided into $\mathrm{CL}_{1}, \mathrm{CL}_{2}$ and $\mathrm{CL}_{3}$ as shown in Fig. 7.

$$
\begin{align*}
& \mathrm{D}(1)_{\mathrm{C}}=\mathrm{D}_{\mathrm{ADD} 1}+\mathrm{D}_{\mathrm{CL} 1}+\mathrm{D}_{\mathrm{CL} 2}  \tag{7}\\
& \mathrm{D}(1)_{\mathrm{S}}=\mathrm{D}_{\mathrm{ADD} 1}+\mathrm{D}_{\mathrm{CL}}+\mathrm{D}_{\mathrm{ADD} 2} \tag{8}
\end{align*}
$$

The delay of the outputs ( $\mathrm{C}_{\text {out }}$ and $\mathrm{dS}_{3: 0}$ ) of the proposed n digit PBA-based XS-3DA can be expressed by formulas (9)(10).

$$
\begin{gather*}
\mathrm{D}(\mathrm{n})_{\mathrm{C}}=\mathrm{D}_{\mathrm{ADD} 1}+\mathrm{D}_{\mathrm{CL} 1}+\mathrm{nD}_{\mathrm{CL} 2}  \tag{9}\\
\mathrm{D}(\mathrm{n})_{\mathrm{S}}=\mathrm{D}_{\mathrm{ADD} 1}+\mathrm{D}_{\mathrm{CL} 1}+\mathrm{nD}_{\mathrm{CL} 2}+\mathrm{D}_{\mathrm{CL} 3}+\mathrm{D}_{\mathrm{ADD} 2} \tag{10}
\end{gather*}
$$

Figure 8 shows the layout of the proposed 1-digit PBAbased XS-3DA implemented in the QCA Designer tool. The proposed 1-digit PBA-based XS-3DA is composed of 715 cells, with an area of $0.68 \mathrm{um}^{2}$, a delay of 3.25 clocks for signal $\mathrm{dS}_{3: 0}$ and a delay of 2.25 clocks for the signal $\mathrm{dC}_{\text {out. }}$. It is a 3-layer design. The low carry input of the leftmost FA of $\mathrm{ADD}_{1}$ is set to " -1 ", and the carry signal $\mathrm{dC}_{\mathrm{in}}$ is directly inputted into the CL circuit and $\mathrm{ADD}_{2}$ for calculation. The proposed designs can use the USE mechanism [13]. The USE mechanism can flexibly create feedback paths of any length and a well-defined clocking circuitry. The clock circuitry of the USE mechanism is simple and the design flexibility is high.


Fig. 8 Layout of the proposed 1-digit PBA-based XS-3DA.

## IV. Simulation and Comparisons

## A. Simulation Results

Figure 9 shows the simulation results of the proposed 1digit PBA-based XS-3DA in the QCA Designer tool. The simulation engine of QCA Designer was set to the coherence vector engine. The other parameters of QCA Designer are set as default. The delay of signal $\mathrm{dC}_{\text {out }}$ is set to the same delay as signal $\mathrm{dS}_{3: 0}$ to synchronize outputs.


Fig. 9. Simulation results of the proposed 1-digit PBA-based XS-3DA.

## B. Complexity Analysis

Table I shows the theoretical comparisons of the proposed XS-3DAs. We estimate area and delay based on the number of used QCA-based gates in the XS-3DAs. $\mathrm{D}_{\text {s }}$ presents the delay of the signal $\mathrm{dS}_{\mathrm{n}-1: \mathrm{n}-4}$ and $\mathrm{D}_{\mathrm{c}}$ presents the delay of the signal $\mathrm{dC}_{\text {out. }}$ Formula (11) presents the calculation method of ADP.

$$
\begin{equation*}
\mathrm{ADP}=\mathrm{Area} \times \mathrm{D}_{\mathrm{s}} \tag{11}
\end{equation*}
$$

The proposed 1-digit RCA-based XS-3DA consists of 16 QCA-based gates, including 7 majority gates, 2 inverters and 7 XOR gates. Thus, the area complexity of the $n$-digit RCAbased XS-3DA is 16 n . The used QCA-based XOR gate requires a delay of a gate for calculation. The QCA-based $\mathrm{ADD}_{1}$ and $\mathrm{ADD}_{2}$ both requires a delay of 4 gates for calculation. The CL circuit of the proposed RCA-based XS-

3DA is composed of an inverter, which requires a delay of a gate. It can be seen from formulas (3)-(4) that, the input signals of the $n$-digit RCA-based adder flow through $n \mathrm{ADD}_{1} \mathrm{~S}$ to get the signal $\mathrm{dCout}_{\text {, }}$, and then flow through one CL circuit and one $\mathrm{ADD}_{2}$ to get the signal $\mathrm{dS}_{\mathrm{n}-1: \mathrm{n}-4}$. Therefore, the signals $\mathrm{dCout}_{\text {out }}$ and $\mathrm{dS}_{\mathrm{n}-1: \mathrm{n}-4}$ of the proposed n-digit RCA-based XS-3DA require a delay of 4 n gates and $4 \mathrm{n}+5$ gates, respectively.
Table I. Theoretical Comparisons of the Proposed XS3DAs

| Design | MG | INV | XOR | Area | Delay |  | ADP |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | $\mathrm{D}_{\mathrm{C}}$ |  |  |
| Pro-RCA1 | 7 | 2 | 7 | 16 | 9 | 4 | 144 |
| Pro-RCAn | 7 n | 2 n | 7 n | 16 n | $4 \mathrm{n}+5$ | 4 n | $64 \mathrm{n}^{2}+80 \mathrm{n}$ |
| Pro-PBA1 | 13 | 1 | 8 | 22 | 13 | 8 | 286 |
| Pro-PBAn | 13 n | n | 8 n | 22 n | $2 \mathrm{n}+11$ | $2 \mathrm{n}+6$ | $44 \mathrm{n}^{2}+242 \mathrm{n}$ |

The proposed 1-digit PBA-based XS-3DA consists of 22 QCA-based gates, including 13 majority gates, 1 inverter and 8 XOR gates. Thus, the area complexity of the n-digit PBAbased XS-3DA is 22 n . The $\mathrm{CL}_{1}, \mathrm{CL}_{2}$ and $\mathrm{CL}_{3}$ of the proposed PBA-based XS-3DA require a delay of two gates, two gates and one gate, respectively. It can be seen from formulas (9)(10) that, the input signals of the $n$-digit PBA-based XS-3DA flow through one $\mathrm{ADD}_{1}$, one $\mathrm{CL}_{1}$ and $\mathrm{n} \mathrm{CL}_{2}$ s to get the signal $\mathrm{dC}_{\text {out }}$, then flow through one $\mathrm{CL}_{3}$ and one $\mathrm{ADD}_{2}$ to get the signal $\mathrm{dS}_{\mathrm{n}-1: \mathrm{n}-4}$. Therefore, the signals $\mathrm{dC}_{\text {out }}$ and $\mathrm{dS}_{\mathrm{n}-1: \mathrm{n}-4}$ of the proposed $n$-digit PBA-based XS-3DA require a delay of $2 n+6$ gates and $2 \mathrm{n}+11$ gates, respectively.

## C. Comparisons

Table II shows the comparison of alternative adders. The proposed multilayer adder has a compact and regular layout, and the positions of the low carry terminal and the high output terminal are easy to access, which is very convenient for the construction of RCA. The proposed adder has a low delay and can be flexibly used in the design of decimal adders.

TABLE II. COMPARISONS OF ALTERNATIVE FAS

| Design | Cell <br> Count | Area <br> $\left(\right.$ um $\left.^{2}\right)$ | Delay <br> (Clocks) | ADP | Layer <br> type | Total <br> energy <br> dissipation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $[10]$ | 86 | 0.09 | 0.75 | 0.068 | M | $2.92 \mathrm{e}-002$ |
| $[11]$ | 58 | 0.03 | 0.75 | 0.023 | M | $2.59 \mathrm{e}-002$ |
| $[12]$ | 57 | 0.04 | 1.00 | 0.040 | C | $1.86 \mathrm{e}-002$ |
| Pro-FA | 57 | 0.05 | 0.50 | 0.025 | M | $2.89 \mathrm{e}-002$ |

Table III shows the comparisons of the decimal adders in terms of cell count, delay, area, ADP and layer type. Based on the proposed two XS-3DAs, 4-digit, 8-digit and 16-digit XS3DAs are constructed, respectively. The decimal adders in [48] are based on 8421 BCD code. It can be seen from Table III that the proposed RCA-based XS-3DAs have the lowest area and delay overhead. The proposed PBA-based XS-3DAs has the lowest area and ADP. Note that, the complexity analysis is based on the number of QCA gates, without considering the QCA wires for synchronous output. With the scale of the designs increases, the delay of the RCA-based XS-3DA begins to be greater than that of the PBA-based XS-3DA, so that the RCA-based XS-3DA requires more QCA wires to synchronize outputs, resulting in the increase of its area. It can be
calculated from Table III that, compared with the 16 -digit RCA-based XS-3DAs, the cell count, area, delay and ADP of the 16 -digit PBA-based XS-3DAs are reduced by $37.88 \%$, $25.99 \%, 37.68 \%$ and $53.88 \%$, respectively.

TABLE III. COMPARISONS OF THE DECIMAL ADDERS

| Design | Digits | Cell <br> Count | Area <br> $\left(\right.$ um $\left.^{2}\right)$ | Delay <br> (Clocks) | ADP | Layer <br> type |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RCA-based | 1 | 669 | 2.28 | 3.00 | 6.84 | M |
| [4] | 4 | 3981 | 11.04 | 8.25 | 91.08 | M |
| $(8421)$ | 8 | 11717 | 34.32 | 15.25 | 523.38 | M |
| RCA-based | 1 | 918 | 3.03 | 4.00 | 12.12 | M |
| [6] | 4 | 4226 | 17.04 | 7.00 | 119.28 | M |
| (8421) | 8 | 10091 | 48.27 | 11.00 | 530.97 | M |
| PBA-based | 1 | 1294 | 2.64 | 3.75 | 9.90 | M |
| [6] | 4 | 5500 | 11.46 | 5.25 | 60.17 | M |
| (8421) | 8 | 13456 | 35.13 | 7.25 | 254.69 | M |
| RCA-based |  |  |  |  |  |  |
| [7] | 1 | 594 | 2.16 | 2.50 | 5.40 | M |
| (8421) |  |  |  |  |  |  |
| RCA-based | 1 | 476 | 0.64 | 2.25 | 1.44 | C |
| [8] | 4 | 3075 | 5.37 | 8.25 | 44.30 | C |
| (8421) | 8 | 9265 | 18.15 | 16.25 | 294.94 | C |
| RCA-based | 1 | 419 | 1.08 | 2.25 | 2.43 | M |
| [8] | 4 | 2178 | 7.56 | 6.75 | 51.03 | M |
| (8421) | 8 | 5710 | 23.88 | 12.75 | 304.47 | M |
| PBA-based | 1 | 733 | 1.01 | 3.50 | 3.54 | C |
| [8] | 4 | 3494 | 4.98 | 6.50 | 32.37 | C |
| (8421) | 8 | 8572 | 12.73 | 10.50 | 133.67 | C |
| PBA-based | 1 | 588 | 1.47 | 3.00 | 4.41 | M |
| [8] | 4 | 2400 | 6.81 | 3.75 | 25.54 | M |
| (8421) | 8 | 5089 | 15.66 | 4.75 | 74.39 | M |
|  | 1 | 539 | 0.60 | 2.25 | 1.35 | M |
| Pro-RCA | 4 | 2771 | 3.16 | 5.25 | 16.59 | M |
| (XS-3) | 8 | 7315 | 9.08 | 9.25 | 83.99 | M |
| Pro-PBA | 4 | 3253 | 3.43 | 4.75 | 16.29 | M |
| (XS-3) | 8 | 7329 | 7.77 | 6.75 | 52.45 | M |
|  | 16 | 17867 | 21.24 | 10.75 | 228.33 | M |
|  |  |  |  |  |  |  |

Table IV. Comparisons of Energy Dissipation of the Proposed XS-3DAs

| Design | Total energy <br> dissipation (eV) | Average energy <br> dissipation $(\mathrm{eV})$ |
| :---: | :---: | :---: |
| Pro- RCA1 | $1.97 \mathrm{e}-001$ | $1.79 \mathrm{e}-002$ |
| Pro- RCA4 | $8.70 \mathrm{e}-001$ | $7.91 \mathrm{e}-002$ |
| Pro- RCA8 | $1.90 \mathrm{e}-000$ | $1.72 \mathrm{e}-001$ |
| Pro- RCA16 | $4.50 \mathrm{e}-000$ | $4.09 \mathrm{e}-001$ |
| Pro- PBA1 | $2.24 \mathrm{e}-001$ | $2.04 \mathrm{e}-002$ |
| Pro- PBA4 | $9.94 \mathrm{e}-001$ | $9.04 \mathrm{e}-002$ |
| Pro- PBA8 | $2.07 \mathrm{e}-000$ | $1.88 \mathrm{e}-001$ |
| Pro- PBA16 | $4.45 \mathrm{e}-000$ | $4.05 \mathrm{e}-001$ |

Table IV shows the comparisons of energy dissipation of the proposed XS-3DAs. The energy dissipation of the proposed designs was estimated by QCA Designer-E tool. Because the CL circuit of the 1-digit PBA-based XS-3DA is more complex, its energy dissipation is greater than that of the 1-digit RCA-based XS-3DA. However, with the design scale
increases, the energy dissipation of PBA-based XS-3DA is gradually less than that of RCA-based XS-3DA (see ProRCA16 vs Pro- PBA16).

## V.Conclusion

In this paper, we have proposed novel RCA-based and PBA-based XS-3DAs in the QCA Designer tool. A new correction circuit is used to construct the PBA-based XS3DAs. Our proposed RCA-based and PBA-based XS-3DAs show excellent performance in terms of area and ADP. We have analyzed and compared the characteristics of the two types of XS-3DAs. The proposed CL circuit can significantly reduce the delay and ADP of the n-digit RCA-based XS-3DAs, thus saving the overall cost of QCA circuit designs.

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