A Practical Approach for the Hybrid Joint-Space Control of Overconstrained Cable-Driven Parallel Robots

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Abstract. Hybrid joint-space control of overconstrained cable-driven parallel robots requires regulating the length of as many cables as the degrees of freedom of the end-effector, and controlling the tension of the remaining ones. Previous studies showed how to choose which cables to length control, and which ones to tension control, according to minimum tension error propagation techniques. The sets of tension-controlled and length-controlled cables achieving minimum tension error propagation typically vary throughout the workspace, which is an undesirable outcome. This paper aims to achieve practicality in hybrid joint-space control, by investigating which cables to be tension- and length-controlled in the whole workspace, without needing to change such sets. The HRPCable robot at LIRMM is analyzed for this application, and experiments are carried out on it.

Keywords: Overconstrained robots · cable-driven parallel robots · force distribution · workspace computation · sensitivity analysis · hybrid joint-space control.

1 Introduction

Overconstrained Cable-Driven Parallel Robots (OCDPRs) employ more cables than those strictly needed to control the end-effector EE degrees of freedom (DoFs). In this way, cables pull against each other, keeping the system under tension and control over a wide range of externally applied loads [4].

For an overconstrained system, determining cable forces is crucial for effectively controlling the EE motion. An optimal set of tensions is usually computed beforehand, solving the force (or tension) distribution (FD) problem based on a chosen criterion, such as minimum tension, maximum stiffness, or others [12]. Lastly, the tensions are converted to servo-drive set-points [15]. If the final objective is the EE pose control, most existing control strategies for OCDPRs imply cascaded control loops [13].

A different strategy was proposed as a simple feed-forward alternative to more complex closed-loop controllers [12], and it was later called hybrid force-position control in joint space (hybrid joint-space control, in short) [10]. According to this strategy, the robot inputs, namely the joint-space controlled variables, are divided
into position- and force-controlled ones, similar to hybrid control techniques in the Cartesian space, where EE controlled variables are similarly divided [7]. In practice, hybrid joint-space control results in several cables equal to the robot degree of redundancy being tension controlled, while the others are length controlled. Since the number of length-controlled cables is equal to the EE DoFs, the pose of the latter is completely determined (if elasticity is negligible).

A selection criterion for force-controlled cables in hybrid joint-space control was initially proposed in [8] and later extended in [10]. This criterion is based on evaluating a performance index, called force-distribution sensitivity to cable tension error (FD sensitivity, in short), which allows quantifying how the FD is influenced by the tension error in a chosen force-controlled cable set. From a practical point of view, the FD sensitivity allows identifying which cable set is best to be force controlled to ensure the lowest error in the overall FD. Usually, there are different zones in the robot workspace characterized by a different optimal cable set. A theoretical minimal cable-tension error would require switching the force-controlled cable set in real time throughout the robot workspace. This feature is unpractical, especially for 8-cable robots, which can have up to 28 different zones [10]; additionally, the switching procedure could cause a discontinuity in the control action and, thus, it could make the robot operation unsafe or jerky.

For hybrid joint-space control purposes, this paper presents a heuristic approach for selecting the force-controlled cable set of an 8-cable OCDPR to keep the cable-tension error as low as possible throughout the WS, without changing the force-controlled cable set. Two practical methods are proposed based on this approach, exploiting the results obtained in controlling a planar 4-cable case [9]. Then, the hybrid joint-space strategy is tested on the HRPCable Robot, an 8-cable OCDPR. The paper is structured as follows. Section 2 introduces the kinetostatic model of a generic OCDPR, and recalls elements of force-distribution sensitivity analysis. In Section 3, the wrench-feasible workspace of the HRPCable robot is characterized, and its force-distribution sensitivity is analyzed. In Section 4, a practical method for selecting the force-controlled cables to be used in hybrid joint-space control is proposed and applied to the HRPCable robot. Section 5 reports the results of some experiments on the application of the proposed methodology on the HRPCable robot. Finally, conclusions are drawn in Sec. 6 and a perspective on future research developments is given.

2 Kinetostatic Modeling

An OCDPR consists of a mobile platform (the EE) connected to a fixed base by \( n > n_d \) cables, which are always under tension. \( n_d \) is the number of EE DoFs, and the degree of redundancy [11] is defined as \( \mu = n - n_d \). Cables are coiled and uncoiled by servo-controlled winches, characterized by a nominally constant transmission ratio. Their length varies proportionally to the actuator rotations if elasticity and hysteresis are neglected [6].

In this work, cables are considered ideal unilateral constraints, thus no mass and elasticity effects are taken into account. According to Fig. 1, \( Oxyz \) is an inertial frame,
whereas \( P'x'y'z' \) is a mobile frame attached to the \( EE \) center of mass. The \( EE \) pose is described by the position vector \( \mathbf{p} \) of \( P \), and a rotation matrix \( \mathbf{R} = \mathbf{R}(\boldsymbol{\epsilon}) \), which describes the orientation of the mobile platform with respect to the base of the mechanism. \( \boldsymbol{\epsilon} \) is a minimal set of orientation parameters, i.e., three Euler's angles. The \( EE \) pose is defined as \( \zeta = [\mathbf{p}^T \, \boldsymbol{\epsilon}^T]^T \).

Cables are attached to the platform, and the fixed base at distal and proximal anchor points \( A_i \) and \( B_i \), respectively, for \( i = 1, \ldots, n \). \( \mathbf{a}_i \) and \( \mathbf{a}'_i \) are the position vectors of \( A_i \) with respect to \( O \) and \( P \) in the inertial frame (Fig 1). The constant position vector of \( A_i \) in the mobile frame is denoted by \( \mathbf{a}'_i \), \( \mathbf{p} \).

The \( i \)-th cable is modeled as the line segment between points \( A_i \) and \( B_i \), and, thus, the \( i \)-th cable vector can be expressed as:

\[
\mathbf{\rho}_i = \mathbf{a}_i - \mathbf{b}_i = \mathbf{p} + \mathbf{R} \mathbf{a}'_i, \mathbf{p} - \mathbf{b}_i
\]  

(1)

Considering \( l_i \) as the \( i \)-th cable length, the constraint imposed by the \( i \)-th cable, and the unit vector of the \( i \)-th cable, pointing from the base towards the platform, are:

\[
\mathbf{\rho}_i^T \mathbf{\rho}_i - l_i^2 = 0,
\quad t_i = \frac{\mathbf{\rho}_i}{l_i}
\]

(2)

The static equilibrium of the mobile platform can be formulated as follows:

\[
-\mathbf{J}^T \mathbf{\tau} - \mathbf{W} = 0,
\quad \mathbf{J}_i = [t_i^T \, t_i^T \, \mathbf{\tilde{a}}_i]
\]

(3)

where \( \mathbf{\tau} \in \mathbb{R}^n \) is the array of cable tensions, \( \mathbf{W} \in \mathbb{R}^{n_d} \) is the external wrench acting on the platform, \( \mathbf{J}^T \in \mathbb{R}^{n_d \times n} \), usually referred to as the structure matrix, is the transpose of the inverse kinematics Jacobian matrix, and \( \mathbf{J}_i \) is the \( i \)-th row of \( \mathbf{J} \) the symbol \( \sim \) over a vector denotes its skew-symmetric representation. For \( OCDPRs (\mu > 0) \), the solution of Eq(3) is underdetermined if all bodies are considered rigid, and infinitely many solutions exist for a given wrench and \( EE \) pose. The solution to this problem consists in computing a tension distribution according to a given criterion [12].
If a hybrid joint-space controller is used, tension errors in force-controlled cables influence the tension in length-controlled cables, and, ultimately, the robot performances. Such influence can be analyzed by computing the force-distribution sensitivity to cable-tension error. Assuming that the tension of the last \( \mu \) cables, denoted by \( \tau_c \), is to be controlled, the array \( \tau \) of cable tensions and the structure matrix can be partitioned as:

\[
\tau \triangleq \begin{bmatrix} \tau_d \\ \tau_c \end{bmatrix}, \quad J^T = \begin{bmatrix} J_d & J_c \end{bmatrix}
\]

where \( J_d \in \mathbb{R}^{n_d \times n_d} \) and \( J_c \in \mathbb{R}^{n_d \times \mu} \), and \( \tau_d \in \mathbb{R}^{n_d} \) denotes the tension of the length-controlled cables. The force-distribution sensitivity \( \sigma \) is defined as the maximum tension variation in the position-controlled cables generated by a unit tension variation (or error) of the force-controlled cables, namely [10]:

\[
\sigma \triangleq \max_{\|\Delta \tau\|_\infty = 1} \| - J_d^{-1} J_c \|_\infty
\]

The \( FD \) sensitivity index can be computed for every set \( j \) of force-controlled cables, thus obtaining \( C \) values, \( \sigma_j \), for \( j = 1, \ldots, C \). \( C \) is the maximum number of ways of selecting \( \mu \) cables out of \( n \), without repetition. After computing all possible \( C \) values of the \( FD \) sensitivity, the minimum value can be identified as:

\[
\sigma^* = \min \sigma_j, \quad j = 1, \ldots, C, \quad C = \binom{n}{\mu} = \frac{n!}{(n - \mu)! \mu!}
\]

The index \( \sigma^* \) points out the cable set that, if force-controlled, propagates the least tension control errors in the other cables. That cable set is denoted by \( j^* \).

### 3 HRPCable Workspace Characterization

In this section, the constant-orientation wrench-feasible WS of an 8-cable OCDPR prototype at LIRMM is characterized with respect to its force-distribution sensitivity to cable-tension error. The prototype, known as HRPCable, was used as an experimental set-up during the EU project Hephaestus [16]. It can be installed in a suspended or overconstrained configuration. For this work, the robot is configured as an OCDPR (Fig.2).

In general, an OCDPR is in a wrench-feasible pose \( \zeta \) if at least one \( FD \) (\( \tau \in \mathbb{R}^n \)) exists, for a given wrench \( W \in \mathbb{R}^{n_d} \), such that the EE equilibrium in Eq. (3) is satisfied with bounded tensions:

\[
\exists \tau: \quad \tau_{min} \leq \tau \leq \tau_{max}, \quad -J^T \tau + W = 0
\]

where \( \tau_{min} \) and \( \tau_{max} \) are the tension limits, and the symbol \( \leq \) denotes element-wise inequality between a scalar and a vector quantity.

The inertial frame \( Oxyz \) is located in the center of the base, and the moving frame \( Px'y'z' \) is at the center of the mobile platform, coinciding with its center of mass. The only external load applied to the robot EE is gravity, and its mass is \( m = 23 \text{kg} \).
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Fig. 2: HRPCable prototype at LIRMM (CNRS - University of Montpellier).

Table 1: Geometrical properties of the HPRCable OCDPR.

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The geometrical properties of the robot are summarised in Tab. 1 and the tension limits are set to \( \tau_{\text{min}} = 30 \text{ N} \), and \( \tau_{\text{max}} = 2000 \text{ N} \). The robot has two degrees of redundancy (\( \mu = 2 \)), and thus 2 cables have to be force-controlled, whereas the others may be length-controlled when a hybrid joint-space strategy is adopted for its control. According to Eq. (6), the number of possible cable combinations is \( C = 28 \).

The minimum FD sensitivity \( \sigma^* \) in the constant-orientation wrench-feasible WS (with \( R = I_3 \), where \( I_3 \) is the identity matrix of order 3) is shown in Fig. 3, whereas Fig. 4 highlights the volumes where the optimal cable set \( j^* \) remains constant. Each color is associated with a certain \( j^* \) pair. The 28 distinct constant-\( j^* \) zones cannot be easily pointed out, and their boundaries are not linear and not easily identifiable.

The analysis of the FD-sensitivity variation for different choices of force-controlled cable pairs is conducted to show how a poor choice of force-controlled cable pair can have negative consequences on robot control. To quantitatively assess how specific cable pairs can be an issue when force-controlled, two thresholds for the sensitivity values are selected: \( \sigma = 2 \text{ N} \) is a practically acceptable value of the minimum sensitivity \( \sigma^* \), whereas \( \sigma = 10 \text{ N} \) is a value where control performances are expected to be appreciably degraded. As an example, if we choose cable pair 2-4 as the force-controlled one (Fig. 5), only in 4% of the WS, the sensitivity value \( \sigma \) is lower or equal to 2N, 30% of the workspace is characterized by 2N < \( \sigma \) < 10N, and
Fig. 3: Variation of the minimum FD sensitivity $\sigma^*$ throughout the wrench-feasible WS of HRPCable OCDPR.

Fig. 4: Wrench-feasible WS of HRPCable characterized by 28 distinct constant $j^*$ zones, where $j^*$ identifies the cable pair to be force-controlled to have the lowest error in the tension distribution in each zone. Each color is associated with a certain $j^*$ pair, and a detailed description of coloring is omitted since it would not add descriptive value.

66% by $\sigma \geq 10N$ (Fig. 5b), with peak values higher than 1000N (Fig. 5a). On the contrary, considering the pair 7-8 (Fig. 6), 68% of the WS poses have sensitivity values lower than 2N, while the others are between 2N and 4.41N, which is the absolute maximum value of $\sigma$ in this case (Fig. 6a). This means that by choosing the pair 7-8 to be force-controlled, the tension error throughout the whole WS would be, at most, around 4.4 times the error in the force-controlled cables.

3 Void zones in the top right corner of Fig. 5a are due to the contour plot nature of the illustration: the FD sensitivity change negligibly in said volume with respect to the rest of the workspace, and contour lines are not depicted.
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Variation of the FD sensitivity when the cable pair 2-4 is force-controlled.

Fig. 5: Variation of the FD sensitivity $\sigma$ for the cable pair 2-4 of HRPCable robot.

Variation of the FD sensitivity when the cable pair 7-8 is force-controlled.

Fig. 6: Variation of the FD sensitivity $\sigma$ for the cable pair 7-8 of HRPCable robot, which gives the best results above all the possible combinations.

4 A practical cable-selection strategy

As highlighted in Sec.3, the constant-$j^*$ map (see Fig.4) derived from the FD-sensitivity analysis is not straightforward to interpret for control purposes. In fact, the constant-$j^*$ zones are numerous ($C = 28$) and they have a highly irregular shape: switching between force- and length-controlled cables while crossing the borders of constant-$j^*$ zones could be very frequent, impractical, and possibly leading to various kinds of control problems, such as control signal discontinuities, vibrations, or unjustifiably large control efforts. A solution to these problems can be found, such as smoothing the control signal with additional low-pass filters to be activated during switching. We propose an alternative that does not require the addition of such control modules, which ultimately may limit the performances due to the induced control lag.
A heuristic method to keep the tension error near the theoretical smallest possible value during the EE motion throughout the WS is introduced. A single cable set can be chosen to be force-controlled in the whole WS. The analysis of several cases showed that to select this cable set, one should evaluate how many points over the totality of the WS have an acceptable value of $\sigma$. We define a value of $\sigma$ as acceptable, if it is less than or equal to the maximum value of the minimum FD sensitivity $\sigma^*(\text{called } \sigma^*_{\text{max}})$ in the WS. $\sigma^*$ is by definition, the minimum value of $\sigma$ for every $j$-th cable set (Eq. (6)). As an example, for the HRPCable $\sigma^*_{\text{max}}$ is equal to 2N.

Considering this criterion, the force-controlled cable set can be chosen by comparing the percentage of WS configurations whose sensitivity is within the acceptable limit ($\sigma \leq \sigma^*_{\text{max}}$) for each cable combination. The best candidate to be force-controlled would be the cable set having the highest percentage of WS configurations within the acceptable sensitivity value. For example, comparing all the cable combinations for the HRPCable robot (the complete analysis is not here reported for the sake of brevity), the pair 7-8 is chosen, as the percentage of FD sensitivity values lower or equal to $\sigma^*_{\text{max}} = 2\text{N}$ is the highest (68%). If this rationale is followed to keep the tension error as low as possible, during the EE motion, one should take into account that in certain WS zones the cable-tension error could be excessively amplified where $\sigma$ has its peaks (e.g., the bottom right corner of the WS, in Fig. 5a), which could limit the robot WS. In other words, the EE should work far from the peaks to avoid tension-error amplification, unless these peaks have a reasonably small value.

Furthermore, for a given force-controlled cable set, the percentage of WS configurations whose sensitivity is within the acceptable limit could be enlarged, optimizing the robot geometry. In fact, from the analysis of the 8-cable cases, it was noticed that a lack of symmetry in the robot structure makes some cable sets more suitable to be force-controlled with respect to others. In a symmetric case, such as the 8-cable IPAnema 3 OCDPR reported in [10], there is no pair significantly better than the others. On the contrary, in the HRPCable case, whose cable attachment points are not symmetric to avoid cable interference (see Tab. 1 and Fig. 2b), the results obtained with the pair 7-8 are clearly better than the results obtained with other pairs, and could be further optimized, for example by allowing the EE to rotate as demonstrated in [8]. Pair 7-8 not only has the highest percentage of configurations with an acceptable $\sigma$ among all cable combinations, but it also shows the lowest maximum value of $\sigma$ in the entire WS (4.41N). Taking this observation into account, it should be possible to optimize the cable anchor point positions in order to enlarge the volume with $\sigma < 2\text{N}$ for this cable set, thus completely removing the need for switching tension- and length-controlled cables.

In general, the analysis reported in this Section highlights that some robot geometries are more effective at being hybrid-controlled than others, and considering this control option at design phase could be more effective than an a-posteriori analysis.

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4 This option will be further analyzed in future studies, but preliminary simulations showed its theoretical feasibility.
5 Experimental Validation

The practical hybrid-input control strategy proposed in this paper is tested on the HRPCable prototype (Fig. 7). Geometrical and inertial parameters of the robot are summarized in Sec. 3, and gravity is the only external wrench acting on the EE. The trajectory planning, the computation of the tension distribution, and the hybrid-input control strategy run on a Beckhoff real-time IPC at 0.5 kHz rate. At the servo-drive level, for ease of implementation, position set-points are always assigned. The $k$-th control update on the $j$-th motor, $p_{j,k}$, is obtained as follows:

$$p_{j,k} = p_{j,k-1} + \Delta p_{j,k}, \quad \forall k, \text{ for } j = 1, \ldots, 8$$ (8)

where $\Delta p_{j,k}$ is computed differently, according to the cable being length- or tension-controlled. In the former case, $\Delta p_{j,k}$ is calculated through inverse kinematics, whereas in the latter case $\Delta p_{j,k} = v_{j,k} \Delta t$, with $\Delta t = 2$ ms and $v_{j,k}$ is assigned, as in Eq. 1 of [15], as the nominal cable velocity to follow the prescribed path.

While the EE orientation is kept such that $R = I_{3 \times 3}$, the same trajectory of the EE reference point is tracked by first controlling the tensions of the cable pair 7-8, which is the best overall, and then by controlling the tensions of the pair 2-4, the worst overall. In both cases, the EE is moved from the home position, $p_H = [0, 0, 0]^T$ m, to the final position $p_F = [-0.2, 0.2, -0.05]^T$ m in 25s. Then, the robot is halted for 10s, and finally goes back to the home position in 25s. Please note that the HRPCable prototype is intended for quasi-static operations, and faster trajectories could not be tested. The tension distribution is computed by applying a minimal 2-norm algorithm with $\tau_{\text{min}} = 30$ N and $\tau_{\text{max}} = 500$ N, that is [3]:

$$\min_\tau \| \tau \|^2 \quad \text{with} \quad \tau_{\text{min}} \leq \tau \leq \tau_{\text{max}}, \quad -J^T \tau - W = 0$$ (9)

Tension error norms, denoted by $\| e_i \| = \| \tau^*_i - \tau_i \|$, with $i = 1, \ldots, 8$, and computed by subtracting the results of Eq. (9) $\tau_i$ to the experimentally obtained values $\tau^*_i$, are
Cable pair 7-8 is tension controlled. (b) Cable pair 2-4 is tension controlled.

Fig. 8: Experimental tension distribution errors.

Fig. 9: FD error 2-norm when cable pairs 7-8, 2-4, and 2-7 are tension controlled.

shown in Fig. 8, the first trajectory from $p_H$ to $p_F$ is executed from $t = 25s$ to $t = 50s$, while the second one from $p_F$ to $p_H$ is executed from $t = 60s$ to $t = 85s$. Errors are generally fairly large due to the robot’s need for maintenance and re-calibration, and the limited time available for experiments was not sufficient for statistical analysis of more trajectories. Nonetheless, Fig. 8a shows that tension-controlling cable pair 7-8 performs generally better, with lower maximum tension errors in several cables, with respect to tension-controlling cable pair 2-4 (Fig. 8b). This result is further confirmed in Fig. 9 by looking at the 2-norm of the FD distribution errors, denoted by $e_{xy} = \|\tau_{xy} - \tau^\star_{xy}\|$ with $xy$ indicating the experiment where the $x$ and $y$ cables were tension-controlled, $\tau$ is the result of Eq. 9, and $\tau^\star$ is the array of the experimental tensions: tension-controlling cable pair 7-8 always results in a smaller 2-norm error than tension-controlling cable pair 2-4. Figure 9 shows an additional result, which was expected in this study: there may be a better cable pair to be locally tension controlled (cable pair 2-7 for the selected trajectory), and the larger error in cable pair 7-8 is the trade-off for not needing to switch the tension-controlled cables throughout the workspace.
6 Conclusions and Outlook

This paper proposed a practical approach for the selection of tension-controlled cables when a hybrid joint-space control technique is used in overconstrained cable-driven parallel robots. The proposed approach is based on the computation of the force-distribution sensitivity to cable-tension errors ($FD_s$ sensitivity). In general, selecting tension-controlled cable pairs through a $FD_s$ sensitivity analysis would require switching the selected cables throughout the workspace, which may be inconvenient. Thus, in this paper, a “best overall” selection strategy that would allow to tension-control always the same cable pair was introduced. This strategy is based on evaluating which tension-controlled cable pair has the largest workspace volume with a force distribution sensitivity below an acceptable threshold. The method was successfully validated through experiments, also showing that the best cable pair overall may not be locally better than other choices. Future studies will focus on optimizing a manipulator $EE$ geometry so that the best cable pair overall is also locally better than any other cable, a result that was hinted at by the analysis proposed in this paper.

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References


