Impact of neuron network on the generation of chaotic attractors

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2022 8th International Conference on Control, Decision and Information Technologies
CoDIT'22 | Istanbul, Turkey / May 17-20, 2022

Impact of neuron network on the generation of chaotic attractors

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***Abstract*— In this paper, new models of chaotic attractors are presented. In doing so, an old chaotic attractor based on the mathematical model of a Variable Structure Model of Neurons (VSMN) is developed. It is then generated by neuron with oscillators. As a result, new styles of new chaotic attractors are found. Also, a detailed bifurcation analysis of those new chaotic systems with theory and simulations is discussed. Finally, the numerical examples will illustrate each model with its simulation using Python. We found out that the new systems are chaotic and could be used for different applications like image encryption.**

Bifurcation, Chaotic attractor, neuron, oscillators, Variable Structure Model of Neurons.

I. INTRODUCTION

Chaos is basic in nonlinear dynamics. It has been strongly studied and adapted in many commercial applications like cryptography [13]. Chaotic systems consist of various basic properties, such as the high sensitivity to the initial conditions and system parameters, topological transitivity, non-periodicity, and pseudo-random properties [16, 11]. Ben Slimane *et al.* [15] proposed a quick and robust scheme for image encryption using the nested chaotic maps and the DNA sequence process. They also designed a strong image crypto system [14] using a single neuron model, a chaotic map and DNA sequence operation. They suggested too, a new technique of chaotic attractors with separated scrolls using the association between the fractal process and the chaotic attractors. Various chaotic systems have been developed, such as Chua circuits, Lorenz attractors and Logistic maps [6]. Bouallegue [5] found a method to generate a new class of chaotic attractors that contain a multi-fractal scroll based on the fractal process. He also proposed a new combination of chaotic attractors with separated scrolls while coupling Julia's process with Chua's attractor and Lorenz's attractor [4]. It is worth knowing that neurons in the work of Bouallegue [3] were used with only one dendrite, we use the same variable structure model of neuron (VSMN) in this work to present new models of chaotic attractors. The structure of this work consists of a single scroll chaotic system presented in Section 2, and simulations of the first, second and third models and their bifurcation diagrams are in section 3, 4 and 5 respectively. Lastly, a conclusion is illustrated in Section 6.

*This work was not supported by any organization

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II. SINGLE SCROLL CHAOTIC SYSTEM

Chaotic behaviour and oscillations have a common factor. They both depend on the mathematical model parameters. Let us present two recurrent equations of oscillators to generate the dynamic of neuron with $k_x = 1/8000 and k_y = 1$:

$$x_{n_2} = 2cos(2\pi k_x)x_{n_1} - x_{n_0}. \quad (1)$$

The second oscillator takes the same form, its used for the second example:

$$y_{n_2} = 2cos(2\pi k_y)y_{n_1} - y_{n_0}. \quad (2)$$

It is an elaboration of the model of neurons presented in [3, 1, 10, 2], but with two dendrites in order to increase the systems precision. The (VSMN) is described by the following model:

$$\begin{cases} \dot{u} = \frac{-(u)}{\tau} + (u+p_1)(u+p_2)f(\beta v)f(\lambda(u+p_1)(u+p_2)) + X \\ \dot{v} = -\alpha v + \chi(u+q_1)^{n_1}(u+q_2)^{n_2}\alpha f^2(\lambda(u+p_1)(u+p_2)). \end{cases} \quad (3)$$

With $X = \rho \sin(2\pi \times f \times t)$.

The structure depends on seven variables n_1, n_2, p_1, p_2, q_1, q_2 and χ. Moreover, u and v are the activity' states of neurons [7]. n_1, n_2, q_1 and q_2 are related to the behavior of dendrites, p_1 and p_2 are related to the positions of dendrites, and χ represents the neurons polarity. Furthermore, Function f(t)=$e^{-t^2/2}$, $k = \pm 1$, τ is a time constant, p and q are real numbers, and α ,β and λ are positive real numbers. Using Python, Fig. 1 shows the implementation result of Eq. 3, its initial conditions are detailed in table 1, with $n_1 = 2$, $n_2 = 2$.

Table 1 also presents the initial conditions used for the first the second and the third model.

TABLE I

INITIAL CONDITIONS OF SINGLE SCROLL CHAOTIC SYSTEM.

u_0	v_0	p_1	p_2	q_1	q_2	α	β	ρ	λ
0.25	0.25	-0.8	0.6	p_1	p_2	0.1	2.667(1+ε)	2.5	3.75

Fig. 1 and Fig. 2 illustrate a spiral chaotic behavior with two orbits connected with a linear straight line. We can call them hidden chaotic attractors, too.

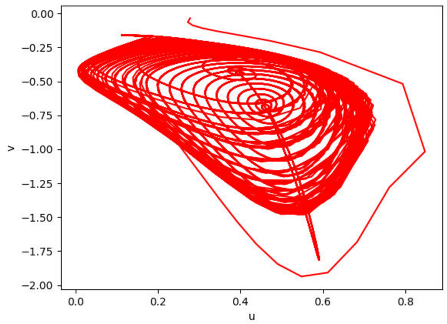

Fig. 1. 2D chaotic system from 0 to 0.8 as abscissa and from 0 to -2 as ordinate.

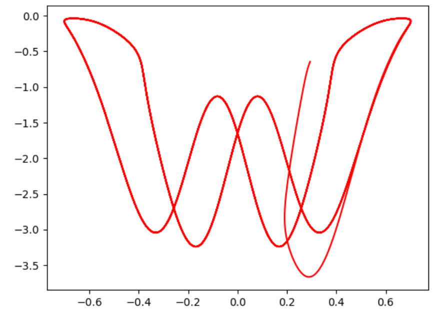

Fig. 3. 2D chaotic system from -0.6 to 0.6 as abscissa and from 0 to -3.5 as ordinate.

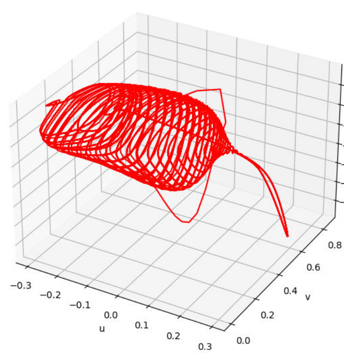

Fig. 2. 3D chaotic system in u-v-w space, from 0 to 0.8 as abscissa then from -0.3 to 0.3 as ordinate and from 0.25 to 1.75 as applicate .

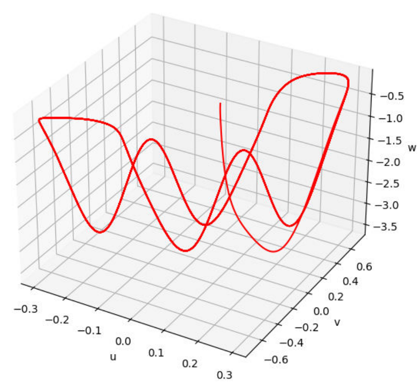

Fig. 4. 3D chaotic system in u-v-w space, from 0 to 0.8 as abscissa then from -0.6 to 0.6 as ordinate and from -3.5 to 3.5 as applicate.

III. FIRST MODEL

A. Simulation of the first model

The first new model is mathematically presented as fellows:

$$\begin{cases} \dot{u} = \frac{-(u)}{\tau} + (u+p_1)(u+p_2)f(\beta v)f(\lambda(u+p_1)(u+p_2)) + 2.5x_{n_2} \\ \dot{v} = -\alpha v + \chi(u+q_1)^{n_1}(u+q_2)^{n_2}\alpha f^2(\lambda(u+p_1)(u+p_2)). \end{cases} \tag{4}$$

We add $2.5 * x_{n_2}$ to the first differential equation $\dot{u}$, which is the first oscillator presented on equation (1). Fig. 3 shows the implementation result of Eq. 4.

Fig. 3 and Fig. 4 illustrate a symmetric behavior connected with a straight line in 2D and 3D phase trajectory respectively, generated by neurons n_1 and n_2.

B. Bifurcation of the first model

In this section, the time-delay influence on the steady state stability is examined, by taking p_1 as bifurcation element and fixing the value of p_2 [8]. The stability of Eq. 4 will vary and roots of periodic orbits will bifurcate from the equilibrium with an increase of p_1. Bifurcation diagram is portrayed in Fig. 5 and it shows the steady state behavior of Eq. 4 for different pairs (p_1, p_2). It can be easily determined that as p_1 increases, then the first period (stable) behavior shown in the color orange is enlarged for greater values of p_1 [9].

Fig. 5 depicts in the color orange the zone of the first period mode and, this meets the greater periods. For good measure, the chaotic zones are also shown as parameters p_1 and p_2 change [12]. We can see from Fig. 5 that the system depicts the chaotic mode which is the black color, from period 0 to 0.3, also from period 0.35 to 0.4 and from period 0.5 to 1.

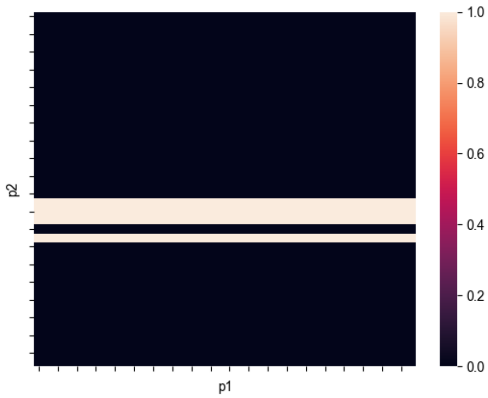

Fig. 5. 2D bifurcation diagram showing steady state behavior of Eq. 4 with different gain pairs (p1, p2).

IV. Second model

A. Simulation of the second model

The second new model is mathematically presented as fellows:

$$\begin{cases} \dot{u} = \frac{-(u)}{\tau} + (u+p_1)(u+p_2)f(\beta v)f(\lambda(u+p_1)(u+p_2)/2) + 2.5x_{n_2} \\ \dot{v} = -\alpha v + \chi(u+q_1)^{n_1}(u+q_2)^{n_2}\alpha f^2(\lambda(u+p_1)(u+p_2)) + 2.5y_{n_2}. \end{cases} \quad (5)$$

Fig. 6 shows the implementation result of Eq.5. Fig. 6 and Fig. 7 present the same chaotic behavior as the first model, with an orbit connected with hidden chaotic attractors. It has a scroll form in the left side of the system.

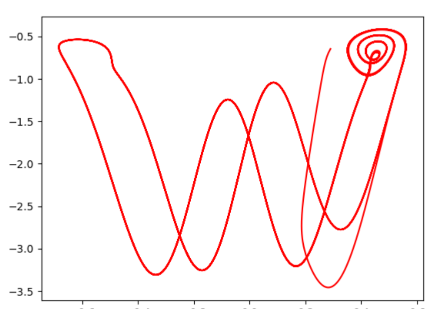

Fig. 6. 2D chaotic system from -0.6 to 0.6 as abscissa and from -3.5 to -0.5 as ordinate.

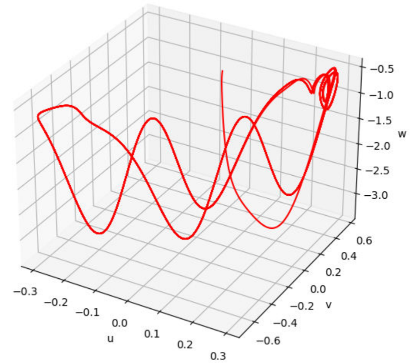

Fig. 7. 3D chaotic system in u-v-w space, from -0.3 to 0.3 as abscissa then from -0.6 to 0.6 as ordinate and from -3 to -0.5 as applicate

B. Bifurcation of the second model

Fig. 8 shows that the system depicts chaotic mode which is the black color, from period 0.39 to 1.

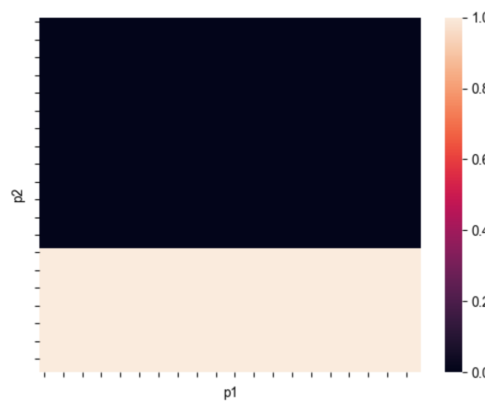

Fig. 8. Bifurcation diagram of the second model.

V. Third model

A. Simulation of the third model

The first new model is mathematically presented as fellows:

$$\begin{cases} \dot{u} = \frac{-(u)}{\tau} + (u+p_1)(u+p_2)f(\beta v)f(\lambda(u+p_1)(u+p_2)/2) + 2.5x_{n_2} \\ \dot{v} = -\alpha v + \chi(u+q_1)^{n_1}(u+q_2)^{n_2}\alpha f^2(\lambda(u+p_1)(u+p_2)). \end{cases} \quad (6)$$

Fig. 9 shows the implementation result of Eq. 6. Fig. 9 and Fig. 10 illustrate a symmetric behavior with two scrolls connected with a linear straight line, one on the right and the other on the right.

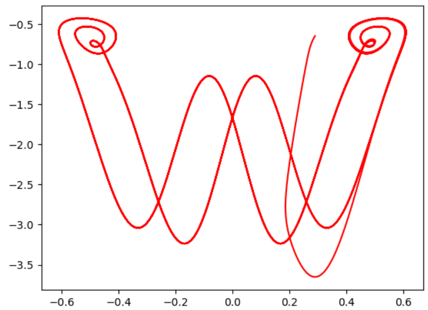

Fig. 9. 2D chaotic system from 0 to 0.8 as abscissa and from 0 to -2 as ordinate.

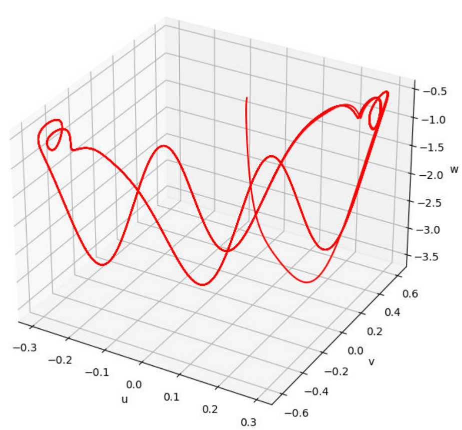

Fig. 10. 3D chaotic system in u-v-w space, from -0.3 to 0.3 as abscissa then from -0.6 to 0.6 as ordinate and from -3.5 to -0.5 as applicate.

B. *Bifurcation of the third model*

Fig. 11 is similar to Fig.5, they inform as that the system depicts the chaotic mode, from period 0 to 0.3, also from from period 0.35 to 0.4 and from period 0.5 to 1.

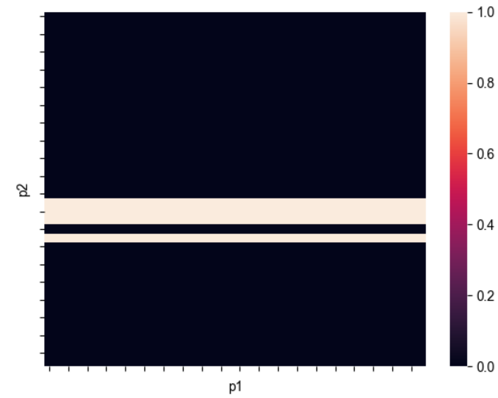

Fig. 11. Bifurcation diagram of the third model

VI. Conclusion

In this work, a generation of chaotic systems using neurons is explored. Those attractors contain separated scrolls. Each scroll looks like a chaotic oscillator. They are all new and different from all the classical approaches. Some numerical simulation present the new models. The most important common element for all of those chaotic generators, is their complexity and their forms. That makes them different and special from the traditional systems. They can ensure better security for image encryption which will be our future contribution.

References

[1] Ghaith Bouallegue, Ridha Djemal, and Kais Belwafi. "Artificial EEG signal generated by a network of neurons with one and two dendrites". In: *Results in Physics* (2020), p. 103699.

[2] Ghaith Bouallegue, Lotfi Hamrouni, and Kais Bouallegue. "Neuron Model With Two Dendrites and its Behaviors". In: *2020 17th International Multi-Conference on Systems, Signals & Devices (SSD)*. IEEE. 2020, pp. 597–603.

[3] Kais Bouallegue. "A new class of neural networks and its applications". In: *Neurocomputing* 249 (2017), pp. 28–47.

[4] Kais Bouallegue. "Chaotic attractors with separated scrolls". In: *Chaos: An Interdisciplinary Journal of Nonlinear Science* 25.7 (2015), p. 073108.

[5] Kais Bouallegue. "Gallery of chaotic attractors generated by fractal network". In: *International Journal of Bifurcation and Chaos* 25.01 (2015), p. 1530002.

[6] Kais Bouallegue, Abdessattar Chaari, and Ahmed Toumi. "Multi-scroll and multi-wing chaotic attractor generated with Julia process fractal". In: *Chaos, solitons & fractals* 44.1-3 (2011), pp. 79–85.

[7] Chunguang Li and Guanrong Chen. "Coexisting chaotic attractors in a single neuron model with adapting feedback synapse". In: *Chaos, Solitons & Fractals* 23.5 (2005), pp. 1599–1604.

[8] Na Li, Wei Tan, and Huitao Zhao. "Hopf Bifurcation Analysis and Chaos Control of a Chaotic System without ilnikov Orbits". In: *Discrete Dynamics in Nature and Society* 2015 (2015).

[9] Yosra Miladi and Moez Feki. "Bifurcation, quasi-periodicity, chaos, and co-existence of different behaviors in the controlled H-bridge inverter". In: *Handbook of Research on Advanced Intelligent Control Engineering and Automation*. IGI Global, 2015, pp. 301–332.

[10] Salah Nasr, Kais Bouallegue, and Hassen Mekki. "Fractal, chaos and neural networks in path generation of mobile robot". In: *International Journal of Modelling, Identification and Control* 34.1 (2020), pp. 41–50.

[11] Salah Nasr, Hassen Mekki, and Kais Bouallegue. "A multi-scroll chaotic system for a higher coverage path planning of a mobile robot using flatness controller". In: *Chaos, Solitons & Fractals* 118 (2019), pp. 366–375.

[12] Bruno Robert, Moez Feki, and Herbert HC Iu. "Control of a pwm inverter using proportional plus extended time-delayed feedback". In: *International Journal of Bifurcation and Chaos* 16.01 (2006), pp. 113–128.

[13] Nabil Ben Slimane, Kais Bouallegue, and Mohsen Machhout. "Designing a multi-scroll chaotic system by operating Logistic map with fractal process". In: *Nonlinear Dynamics* 88.3 (2017), pp. 1655–1675.

[14] Nabil Ben Slimane et al. "A novel chaotic image cryptosystem based on DNA sequence operations and single neuron model". In: *Multimedia Tools and Applications* 77.23 (2018), pp. 30993–31019.

[15] Nabil Ben Slimane et al. "An efficient nested chaotic image encryption algorithm based on DNA sequence". In: *International Journal of Modern Physics C* 29.07 (2018), p. 1850058.

[16] Nabil Ben Slimane et al. "Hash key-based image cryptosystem using chaotic maps and cellular automata". In: *2018 15th International Multi-Conference on Systems, Signals & Devices (SSD)*. IEEE. 2018, pp. 190–194.