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Relational Concept Analysis: Where Formal Concepts Meet Description Logics

Marianne Huchard
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Journées Mondiales de la Logique 2025

Introduction

Formal Concept Analysis & Propositional Logic

Relational Concept Analysis & Description Logics

RCA -DL opportunities

Concluding remarks

Agenda

Introduction

Formal Concept Analysis & Propositional Logic

Relational Concept Analysis & Description Logics

RCA -DL opportunities

Concluding remarks

Ingredients

Formal concepts are “a natural feature of information representation which is as fundamental to hierarchies and object/attribute structures as set theory or relational algebra are for relational databases”.

Uta Priss. 40th Anniv. Vol. of Annual Review of Inf. Sc. and Tech., 2006



Description Logics are “a family of knowledge representation languages that can be used to represent the knowledge of an application domain in a structured and formally well-understood way. The name description logics is motivated by the fact that [...] the important notions of the domain are described by **concept descriptions**”.



F. Baader, I. Horrocks, U. Sattler. Description Logics, Foundations of Artificial Intelligence, Chap. 3, Vol. 3, Elsevier, 2008

Agenda

Introduction

Formal Concept Analysis & Propositional Logic

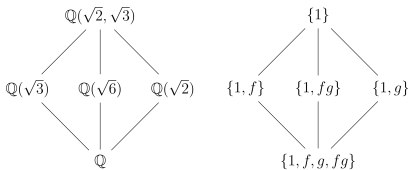
Relational Concept Analysis & Description Logics

RCA -DL opportunities

Concluding remarks

Formal Concept Analysis: Emergence of a research field

Galois correspondences *in* Algebra (1830)



Wikipedia: [Evariste Galois \(1811-1932\)](#); [The last page of its last manuscript](#); [A Duel in the Bois De Boulogne, Near Paris, wood-engraving after G. Durand, Harper's Weekly \(Jan. 1875\)](#); [H. Lecomte, Combat de la rue de Rohan le 29 juillet 1830](#);

The Galois Fundamental theorem "asserts that given a field extension E/F that is finite and Galois, there is a one-to-one correspondence between its intermediate fields and subgroups of its Galois group." (wikipedia) Ex. of Galois correspondence "Lattice of subfields (left) for K/\mathbb{Q} and inverted Lattice of subgroups of $\text{Gal}(K/\mathbb{Q})$ "(wikipedia)

Formal Concept Analysis: Emergence of a research field

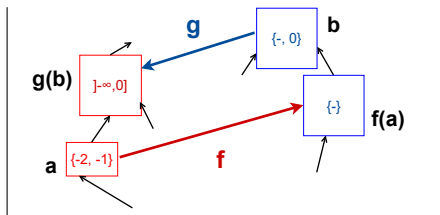
Galois connection *in* partial order and lattice theory (1940-1944)



Georges David Birkhoff, 1940; Øystein Ore, 1944

A **monotone Galois connection** between (A, \leq) and (B, \leq) is a pair (f, g) s.t. f, g monotone and $f : A \rightarrow B$ and $g : B \rightarrow A \forall a \in A, b \in B,$
 $a \leq g(b) \Leftrightarrow f(a) \leq b$

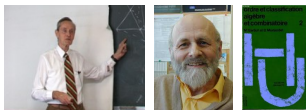
(A, \leq)
Sets of integers
ordered by
inclusion



(B, \leq)
Set of signs
ordered by
inclusion

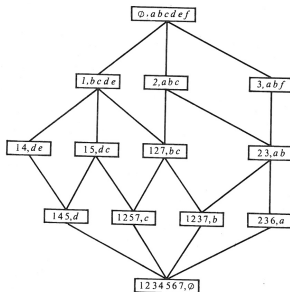
Formal Concept Analysis: Emergence of a research field

Galois lattices *in* math. & social sciences (1970)



Highlighting of the Galois connection associated with a binary relation
Application to qualitative multivariate analysis of questionnaires

$E \setminus E'$	1	2	3	4	5	6	7
a		1	1			1	
b	1	1	1				1
c	1	1			1		1
d	1			1	1		
e	1			1			
f			1				



1982: Restructuring Lattice Theory (Rudolf Wille)

RESTRUCTURING LATTICE THEORY:
AN APPROACH BASED ON HIERARCHIES OF CONCEPTS

Rudolf Wille
Fachbereich Mathematik
Technische Hochschule Darmstadt
6100 Darmstadt
Federal Republic of Germany

ABSTRACT

Lattice theory today reflects the general status of current mathematics: there is a rich production of theoretical concepts, results, and developments, many of which are reached by elaborate mental gymnastics; on the other hand, the connections of the theory to its surroundings are getting weaker and weaker, with the result that the theory and even many of its parts become more isolated. Restructuring lattice theory is an attempt to reinvigorate connections with our general culture by interpreting the theory as concretely as possible, and in this way to promote better communication between lattice theorists and potential users of lattice theory.

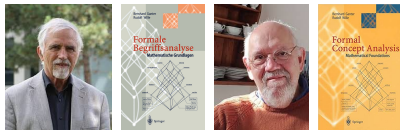
The approach reported here goes back to the origin of the lattice concept in nineteenth-century attempts to formalize logic, where a fundamental step was the reduction of a concept to its "extent". We propose to make the reduction less abstract by retaining in some measure the "intent" of a concept. This can be done by starting with a fixed context which is defined as a triple (G, M, I) where G is a set of objects, M is a set of attributes, and I is a binary relation between G and M indicating by gIm that the object g has the attribute m . There is a natural Galois connection between G and M defined by $A' = \{m \in M \mid gIm \text{ for all } g \in A\}$ for $A \subseteq G$ and $B' = \{g \in G \mid gIm \text{ for all } m \in B\}$ for $B \subseteq M$. Now, a concept of the context (G, M, I) is introduced as a pair (A, B) with $A \subseteq G$, $B \subseteq M$, $A' = B$, and $B' = A$, where A is called the extent and B the intent of the concept (A, B) . The hierarchy of concepts given by the relation subconcept-superconcept is captured by the definition $(A_1, B_1) \sqsubseteq (A_2, B_2) \Leftrightarrow A_1 \subseteq A_2 (\Leftrightarrow B_1 \supseteq B_2)$ for concepts (A_1, B_1) and (A_2, B_2) of (G, M, I) . Let $L(G, M, I)$ be the

445

I. Rival (ed.), Ordered Sets, 445–470.
Copyright © 1982 by D. Reidel Publishing Company.



Formal Concept Analysis: Emergence of a research field



Bernhard Ganter & Rudolf Wille, Seminal book 1999

- Philosophical perspective, Spirit of Port-Royal terminology (Arnauld et Nicole, 1683)
- Terminology: Formal concepts, Extension/Intension (extent/intent), Concept lattice
- Many theoretical tools including the logical perspective
- Opening towards applications

Formal Concept Analysis

Simple but powerful basics

- Formal Context
- Galois connection
- Concept Lattice
- Connections with propositional logic

Formal Context = Triple (O, A, R)

O is a finite set of objects, A is a finite set of attributes

$R \subseteq O \times A$ is a binary relation

$(o, a) \in R$ means that object o owns attribute a .

Ingredient	<i>vege</i>	<i>vegan</i>	<i>spring</i>	<i>summer</i>	<i>autumn</i>
goatcheese	×			×	
burrata	×		×		
scallop			×		
tomato	×	×		×	
shallot	×	×			×
mushroom	×	×			×
eggplant	×	×		×	

Galois connection and formal concepts

Two operators, both denoted by $'$, connect the powerset of objects, 2^O and the powerset of attributes 2^A as follows:

$$' : 2^O \rightarrow 2^A, X' = \{a \in A \mid \forall o \in X, (o, a) \in R\}$$

$$' : 2^A \rightarrow 2^O, Y' = \{o \in O \mid \forall a \in Y, (o, a) \in R\}$$

The pair of $'$ operators induces a Galois connection between 2^O and 2^A

A pair (X, Y) where $X \in 2^O$, $Y \in 2^A$, $X = Y'$, and $Y = X'$, is a (*formal*) *concept*, with X as *extent* and Y as *intent*.

Formal Concept = Pair (Extent, Intent)

Extent = Maximal set of owner objects

Intent = Maximal set of their shared attributes

Ingredient	<i>vege</i>	<i>vegan</i>	<i>spring</i>	<i>summer</i>	<i>autumn</i>
goatcheese	×			×	
burrata	×		×		
scallop			×		
tomato	×	×		×	
shallot	×	×			×
mushroom	×	×			×
eggplant	×	×		×	

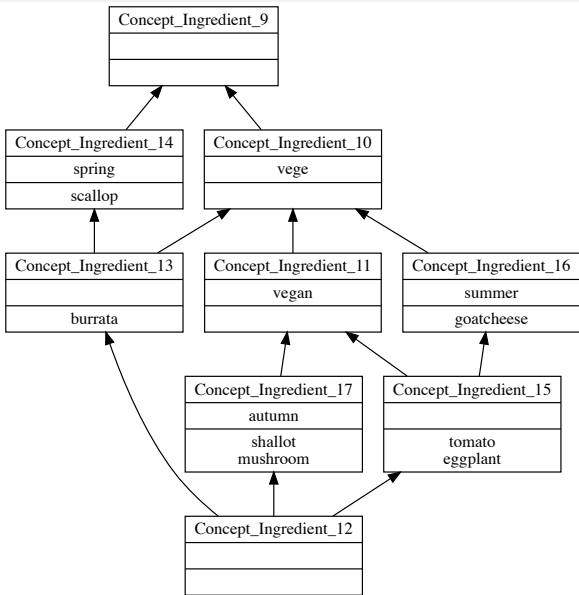
Concept lattices

The set $\mathcal{C}_{\mathcal{K}}$ of all concepts extracted from \mathcal{K} ordered by extent inclusion forms a complete lattice, $\mathcal{L}_{\mathcal{K}} = \langle \mathcal{C}_{\mathcal{K}}, \leq_{\mathcal{K}} \rangle$, called the *concept lattice* of the context (or the *Galois lattice* of the binary relation R)

Concept Lattice

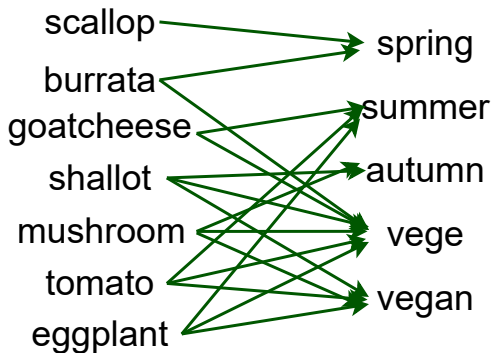
Set of all concepts provided with extent \uparrow or intent \downarrow inclusion

C_I_11
vege
vegan
shallot
mushroom
tomato
eggplant



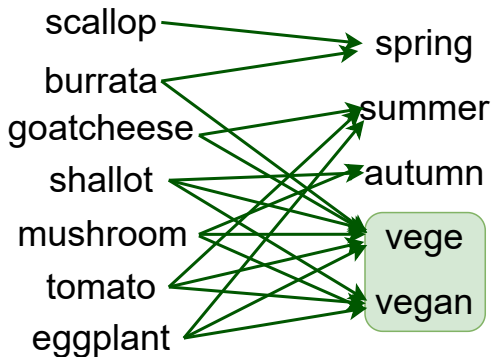
Another view of concept building

Links between objects and attributes



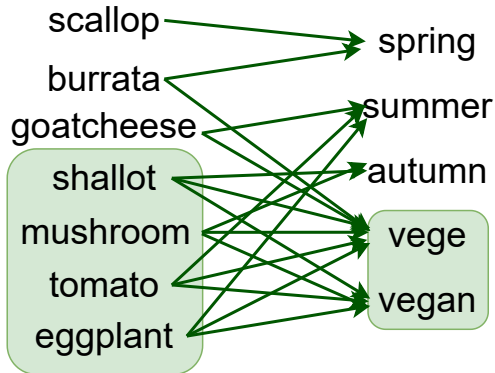
Another view of concept building

Links between objects and attributes *vege / vegan*



Another view of concept building

Concept of **vegan ingredients**



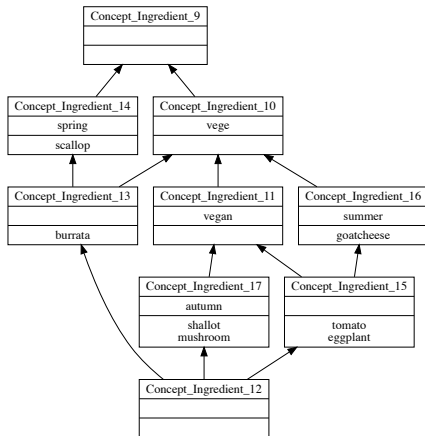
Connections with propositional logic

“One of the aspects of FCA thus is **attribute logic**, the study of possible attribute combinations. Most of the time, this will be very elementary. Those with a background in Mathematical Logic might say that attribute logic is just Propositional Calculus, and thus Boolean Logic, or even a fragment of this. Historically, the name Propositional Logic is misleading: Boole himself used the intuition of attributes (signs) rather than of propositions. So in fact, attribute logic goes back to Boole. But our style is different from that of logicians. Our logic is contextual, which means that **we are interested in the logical structure of concrete data (of the context)**. Of course, the general rules of mathematical logic are important for this and will be utilized.” B. Ganter.

https://www.kde.cs.uni-kassel.de/lehre/ss2005/formale_begriffsanalyse/skript/chapter1_2.pdf

- Constraints
- Implications, association rules

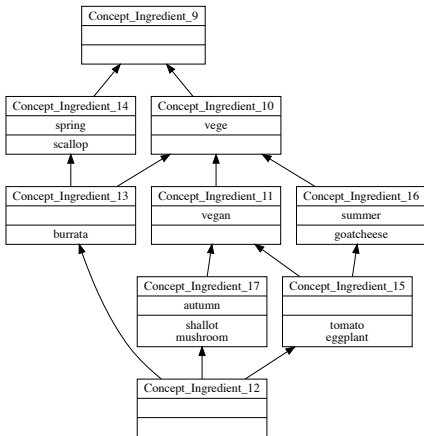
Logical Constraints: Mutual exclusion



Intersection of C14 and C11 extents is empty

$$\neg(\textit{spring} \wedge \textit{vegan})$$

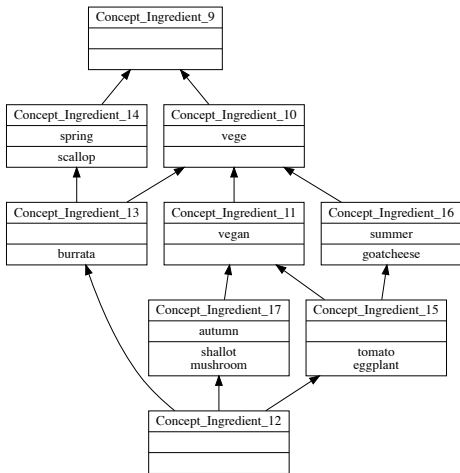
Logical Constraints: Or



Extent of C9 is covered by extents of C14 and C10
(possibly non disjoint)

Ingredients verify: $spring \vee vege$

Logical Constraints: Xor



Extents of C14, C17 and C16 form a partition of C9 Extent
Ingredients verify: $spring \oplus autumn \oplus summer$

Implication rules $Prem \implies Conc$ of $K = (O, A, R)$

Pair $(Prem, Conc) \subseteq A \times A$ s.t. $g(Prem) \subseteq g(Conc)$

all the *objects* that own the *attributes* of *Prem* (premise) also own the *attributes* of *Conc* (conclusion)

vegan \implies vege

Implication rules

Criteria for an implication set

- **Sound**: any implication of the set holds
- **Complete**: any other implication can be obtained from the set
- **Basis**: sound and complete
- **Minimal cardinality**
- **Direct**: any attribute closed set can be obtained in one iteration
- **No redundancy** between rules and inside rules

Diverse approaches

- Minimal non-redundant set of binary implications
- Basis of Duquennes-Guigues [Guigues, 1986] (For a systematic study, see [Bertet&Monjardet, 2010])
- Left-minimal direct basis of implications [Cordero 2013]
- Basis of proper premises [Reppe, 2008; Ryssel, 2014]
- and others ...

Duquenne-Guigues Basis of Implications (DGBI)

“What we can remember and gather from the data forgetting the objects” V. Duquenne, 1987



Cardinality minimal set of non redundant implications

<support> implication

support = number of objects that hold the implication

<2> autumn => vege,vegan

<3> summer => vege

<4> vegan => vege

<0> vege,spring,summer => vegan,autumn

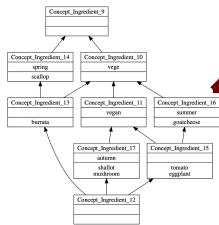
<0> vege,vegan,summer,autumn => spring

<0> vege,vegan,spring => summer,autumn

Nota: redundancy can occur in a premiss or in a conclusion

Magic triangle

Ingredient	vege	vegan	spring	summer	autumn
goatcheese	x			x	
burrata	x		x		
scallop			x		
tomato	x	x		x	
shallot	x	x			x
mushroom	x	x			x
eggplant	x	x		x	



<2> autumn => vege,vegan

<3> summer => vege

<4> vegan => vege

<0> vege,spring,summer => vegan,autumn

<0> vege,vegan,summer,autumn => spring

<0> vege,vegan,spring => summer,autumn

The magic triangle of FCA

Limits and developments



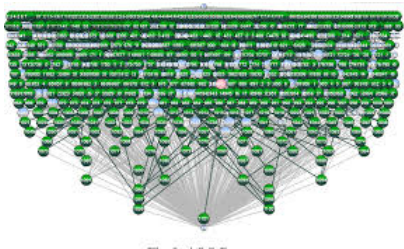
Non tabular
data



Imperfect data



Deluge of
patterns



Huge lattice



“what kind of
alien really
reads Figure
3 concept
lattice...”

Complex data

- **Multi-valued attributes** : integers, double, terms, structures, symbolic objects, etc. (Ganter et Wille, Diday, Polaillon, ...)
- **Fuzzy** (Belohlavek et al., Cabrera, Cordero, Enciso, Mora, Lòpez-Ròdriguez, Ojeda-Aciego et al., Cornejo, Medina et al., Yahia et al., Dubois, Prade ...)
- **Value taxonomies** (Godin et al., Carpineto et Romano, ...)
- **Logical description** (Chaudron et al., Ferré et al., ...)
- **Graphs** (Ganter and Kuznetsov, Liquière, Prediger et Wille, Kötters et al., Graph-FCA Ferré et al....)
- **Multi-relational, RCA** (Priss, Rouane et al., ...); **RCA+Fuzzy** (Boffa et al.)
- **Polyadic** (Sacarea, Tronca et al.)
- **Sequences** (Boukhetta, Demko, Bertet et al., Buzmakov et al.)
- **Temporal** data (Wolff et al., Nica, Braud, Dolques, Le Ber et al., Boukhetta, Demko, Bertet et al.)
- **Pattern Structures** (Ganter et al., Kuznetsov et al., Napoli et al., Buzmakov et al.)

Agenda

Introduction

Formal Concept Analysis & Propositional Logic

Relational Concept Analysis & Description Logics

RCA -DL opportunities

Concluding remarks

Basics of Relational Concept Analysis

Principles

- Extends the purpose of FCA for taking into account object categories and links between objects
- Main principles:
 - A relational model based on the entity-relationship model (restricted to unidirectional binary relationships)
 - *relational attributes* integrate relations in formal contexts between objects
 - Various operators (*quantifiers*) inspired by description logics
 - Iterative and tunable process
- RCA provides a *set of interconnected lattices*
- Translation in the Description logics framework

M. Huchard, M. A. Rouane-Hacène, C. Roume, P. Valtchev: Relational concept discovery in structured datasets. . Ann. Math. Artif. Intell. 2007

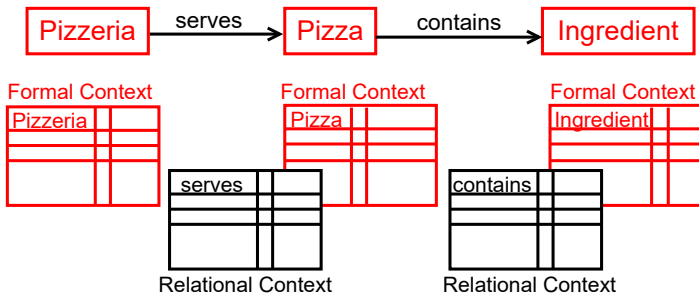
M. A. Rouane-Hacene, M. Huchard, A. Napoli, P. Valtchev: Relational concept analysis: mining concept lattices from multi-relational data. . Ann. Math. Artif. Intell. 2013



Principles: Input data

Relational Context Family (RCF)

- Set of Formal contexts (object-attribute)
 - Represent classes/instances
- Set of Relational contexts (object-object)
 - Represent Unidirectional binary associations/links



Relational Context Family (RCF)

A *relational context family* \mathcal{R} is a pair (\mathbf{K}, \mathbf{R}) , where:

- \mathbf{K} denotes the set of formal contexts $\mathcal{K}_i = (O_i, A_i, I_i)$,
- \mathbf{R} denotes the set of relational contexts $r_k \subseteq O_i \times O_j$, where O_i and O_j are the object sets of formal contexts \mathcal{K}_i and \mathcal{K}_j .

Relational Context Family Pizzerias

Pizzeria	<i>ldHappizy</i>	<i>ldEataly</i>	<i>ldLafelicita</i>	<i>ldSmallitaly</i>
<i>happizy</i>	x			
<i>eataly</i>		x		
<i>lafelicita</i>			x	
<i>smallitaly</i>			x	

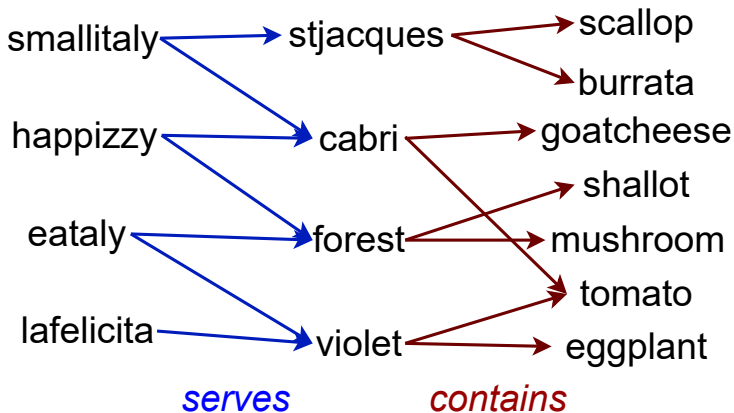
Pizza	<i>ldCabri</i>	<i>ldForest</i>	<i>ldViolet</i>	<i>ldStjacques</i>
<i>cabri</i>	x			
<i>forest</i>		x		
<i>violet</i>			x	
<i>stjacques</i>			x	

Ingredient	<i>vege</i>	<i>vegan</i>	<i>spring</i>	<i>summer</i>	<i>autumn</i>
<i>goatcheese</i>	x			x	
<i>burrata</i>	x		x		
<i>scallop</i>			x		
<i>tomato</i>	x	x		x	
<i>shallot</i>	x	x			x
<i>mushroom</i>	x	x			x
<i>eggplant</i>	x	x		x	

contains	<i>goatcheese</i>	<i>burrata</i>	<i>scallop</i>	<i>tomato</i>	<i>shallot</i>	<i>mushroom</i>	<i>eggplant</i>
<i>cabri</i>	x			x			
<i>forest</i>					x	x	
<i>violet</i>				x			x
<i>stjacques</i>		x	x				

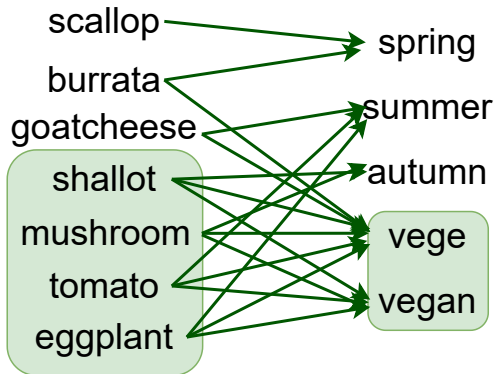
serves	<i>cabri</i>	<i>forest</i>	<i>violet</i>	<i>stjacques</i>
<i>happizy</i>	x	x		
<i>eataly</i>		x	x	
<i>lafelicita</i>			x	
<i>smallitaly</i>	x			x

Concept building



Vegan ingredients

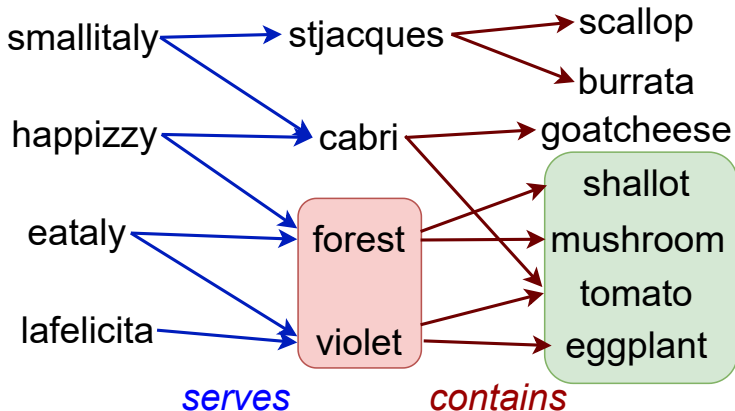
Concept of **vegan ingredients**



Concept building: relational attribute introduction

Concept pizzas that have all their ingredients in
concept vegan ingredients

forest and violet share: $\exists \vee \text{contains}$ (Vegan)



Concept building: relational attribute introduction

Objective: consistently introduce in FCA framework abstractions similar to the role restrictions $\forall r.C$ or $\exists r.C$ in DL

Role restrictions provide summaries of object links:

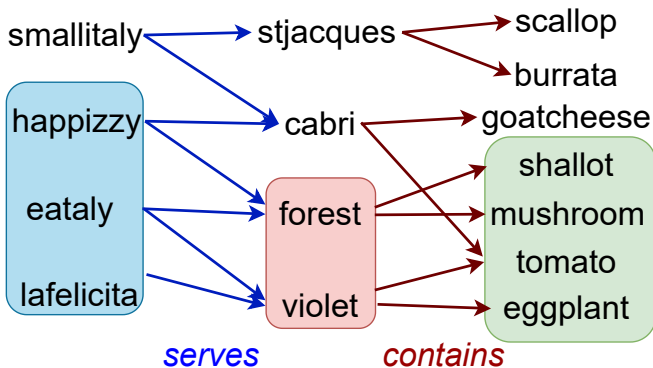
- $\exists r.C$ expresses that for any object satisfying the restriction, at least one of its “ r links” points to instances of C .
- $\forall r.C$ expresses that for any object satisfying the restriction, all its “ r links” point to instances of C .
- $\exists\forall r.C$ expresses that for any object satisfying the restriction, at least one and all its “ r links” point to instances of C .

Concept building

Concept **pizzerias** that have **at least one pizza** in concept **pizzas** that have **all their ingredients** in concept **vegan ingredients**

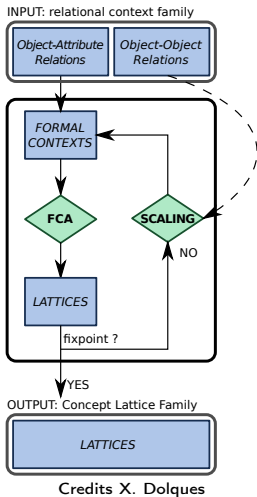
happizzy, eataly and lafelicitita share:

\exists *serves* ($\exists \forall$ *contains* (*Vegan*))

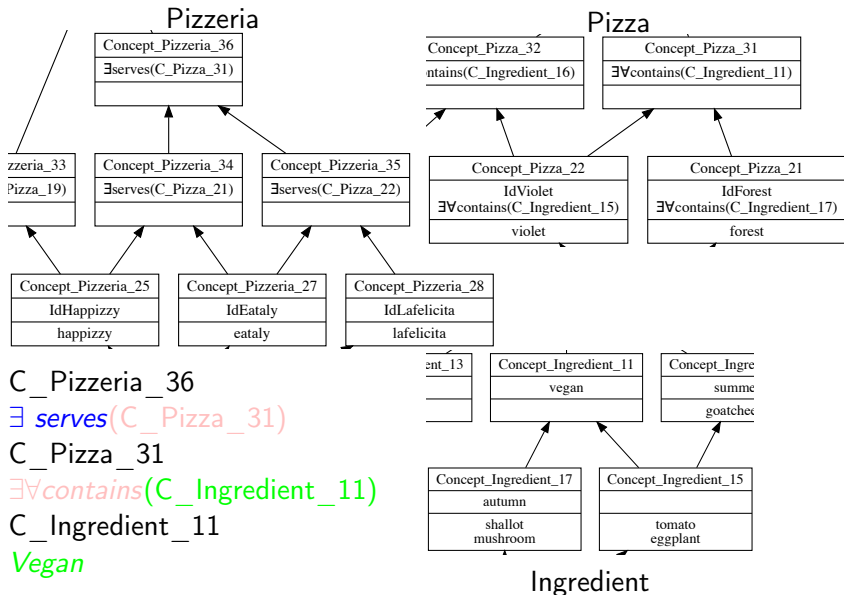


RCA process

- Initialization
 - Build the concept lattice associated with each formal context
- Iteration until a fix point is reached
 - Scaling: Extend each formal context FC with the relational attributes associated with all relations that have FC as domain, for chosen quantifiers.
 - Build the concept lattice associated with each extended formal context



Interconnected concept lattices

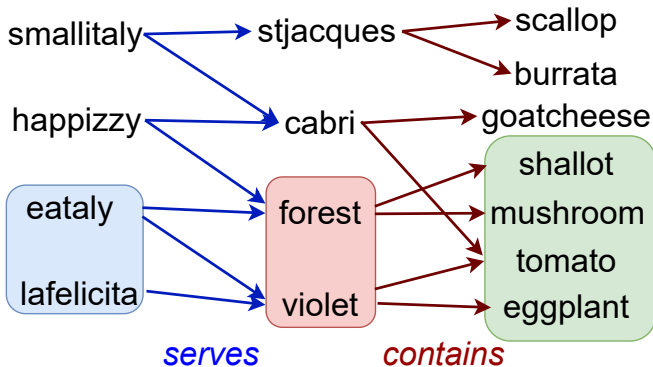


Concept building

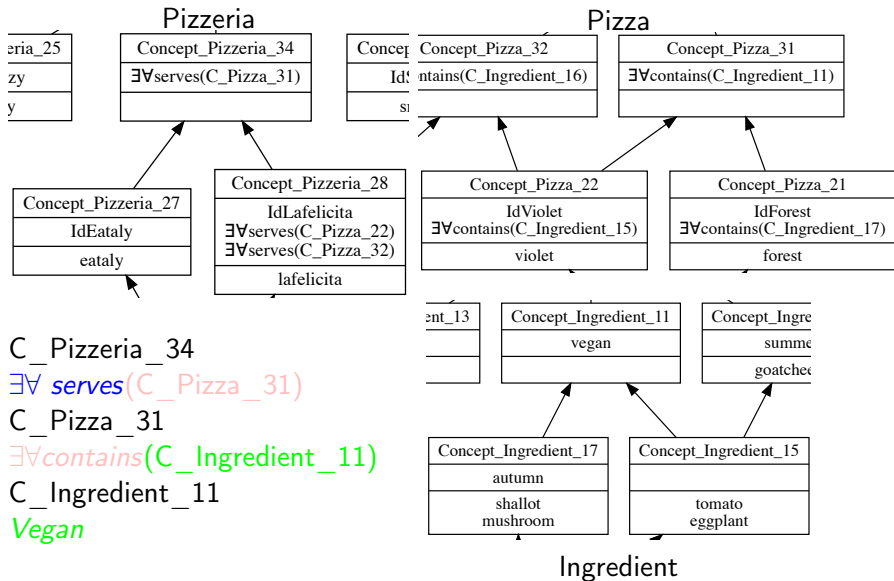
Concept **pizzerias** that have **all** their pizzas in concept **pizzas** that have **all** their ingredients in concept **vegan ingredients**

eataly and lafelicita share:

\exists *serves* (\exists *contains* (*Vegan*))

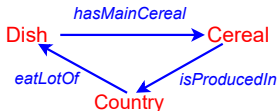


Interconnected concept lattices

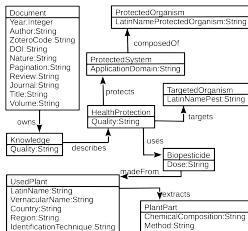


Aspects of RCA

Cyclic models
(including cyclic data)



Complex models



Credits P. Martin

A variety of quantifiers (excerpt)

Operator	Attribute form	Condition
Universal (narrow)	$\forall r.c$	$r(o) \subseteq Ext(c)$
Universal strict	$\forall \exists r.c$	$r(o) \subseteq Ext(c)$ and $r(o) \neq \emptyset$
Universal-percent	$\forall \exists \geq n\% r.c$	$ r(o) \cap Extent(C) \geq n r(o) /100$
Covers	$\supseteq r.c$	$r(o) \supseteq Ext(c)$
Covers-percent	$\supseteq \geq n\% r.c$	$ r(o) \cap Extent(C) \geq n Extent(C) /100$
Existential (wide)	$\exists r.c$	$r(o) \cap Ext(c) \neq \emptyset$
Universal strict	$\forall \exists r.c$	$r(o) \subseteq Ext(c)$ and $r(o) \neq \emptyset$
Qualif. card. restriction	$\geq n r.c$	$r(o) \subseteq Ext(c)$ and $ r(o) \geq n$
Card. restriction	$\geq n r.T_{\mathcal{L}}$	$ r(o) \geq n$

Basics of description logics

Family of knowledge representation formalisms, where an application domain is described in terms of concepts, roles and individuals

Knowledge basis:

- TBox (terms: concepts and relations)
- ABox (assertions)

Syntax excerpt:

syntax	constructor	syntax	constructor
\top	universal	\perp	absurd
$c1 \sqcap c2$	conjunction	$c1 \sqcup c2$	disjunction
$c1 \sqsubseteq c2$	concept subsumption	$r1 \sqsubseteq r2$	role subsumption
$\forall r.c$	universal restriction	$\exists r.c$	existential restriction
$(\geq n \ r)$	cardinality sup restriction	$(\leq n \ r)$	cardinality inf restriction
$\neg c1$	concept negation	$r1 \wedge r2$	role composition

F. Baader, D. Calvanese, D. L. McGuinness, D. Nardi, P. F. Patel-Schneider: The Description Logic Handbook: Theory, Implementation, Applications. Cambridge University Press, Cambridge, UK, 2003.

Connection RCA - DL ($\mathcal{FL}\mathcal{E}^+$)

A bijective mapping α associates a DL construct, of TBox and ABox ($\mathcal{T}(\mathcal{T}_C, \mathcal{T}_R), \mathcal{A}$) in $\mathcal{FL}\mathcal{E}^+$: **conjunction, universal and existential restrictions** to each RCA entity

Construction of the TBox

Source rca entity	Notation	Target dl element	Rule
context	$\mathcal{K} \in \mathbf{K}$	primitive concept	$\alpha[\mathcal{K}] \in \mathcal{T}_C$
atomic attribute	$a \in A_i$	primitive concept	$\alpha[a] \in \mathcal{T}_C$
relation	$r \in \mathbf{R}$	primitive role	$\alpha[r] \in \mathcal{T}_R$
relational attribute (narrow)	$\forall r(c) \in A_i^\infty$	value restriction	$\forall \alpha[r].\alpha[c] \in \mathcal{T}$
relational attribute (wide)	$\exists r(c) \in A_i^\infty$	existential role quantification	$\exists \alpha[r].\alpha[c] \in \mathcal{T}$
concept	$c \in \mathcal{L}_i^\infty$, $c \neq \top_{\mathcal{L}_i^\infty}$	defined concept	$\alpha[c] \in \text{definitions}(\mathcal{T})$
		concept definition	$(\alpha[c] \equiv \prod_{a_j \in \text{int}(c)} \alpha[a_j]) \in \mathcal{T}$
		inclusion axiom	$(\alpha[c] \sqsubseteq \alpha[\mathcal{K}_i]) \in \mathcal{T}$
sub-concept link	$c_1 \leq_{\mathcal{L}_i^\infty} c_2$	inclusion axiom	$(\alpha[c_1] \sqsubseteq \alpha[c_2]) \in \mathcal{T}$

Connection RCA - DL

A bijective mapping α associates a DL construct to each RCA entity

Construction of the TBox (examples)

Primitive concepts

- Ingredient, Pizza, Pizzeria, Vege, Vegan, ..., Autumn

Primitive roles

- serves, contains

Defined concepts

- $C_Pizzeria_34 \equiv \exists \forall serves (C_Pizza_31)$
- $C_Pizza_31 \equiv \exists \forall serves (C_Ingredient_11)$
- $C_Ingredient_11 \equiv Ingredient \sqcap Vegan$
- $C_Ingredient_17 \equiv Ingredient \sqcap Vegan \sqcap Autumn$

Connection RCA - DL

A bijective mapping α associates a DL construct to each RCA entity

Construction of the ABox

Source rca entity	Notation	Target dl element	Rule
objects	$o \in O_i$	individual	$\alpha[o] \in \text{individuals}(\mathcal{A})$
		primitive concept instantiation	$\alpha[\mathcal{K}_i](\alpha[o]) \in \mathcal{A}$
object-to-attribute incidence	$(o, a) \in I_j^0$	primitive concept instantiation	$\alpha[a](\alpha[o]) \in \mathcal{A}$
relational link	$r(o_1, o_2), r_i \in \mathbf{R}$	role instance	$\alpha[r](\alpha[o_1], \alpha[o_2]) \in \mathcal{A}$
object-to-concept incidence	$c \in \mathcal{L}_i^\infty, o \in \text{ext}(c)$	defined concept instantiation	$\alpha[c](\alpha[o]) \in \mathcal{A}$

Connection RCA - DL

A bijective mapping α associates a DL construct to each RCA entity

Construction of the ABox (examples)

- Individuals: eataly, violet, shallot, eggplant
- Primitive concept instantiation: Vegan(shallot)
- Role instances: serves(eataly,violet), contains(violet,eggplant)
- Defined concept instantiation: C_Pizzeria_34(eataly),
C_Pizza_31(violet), C_Ingredient_11(shallot),
C_Ingredient_17(shallot)

Connection RCA - DL

Each extended formal context has its associated implications.
E.g. for RCA with \exists *serves*, $\exists\forall$ *contains* setting, a few implications for pizzerias:

$\langle 2 \rangle \exists \text{ serves}(\text{IdViolet} \& \exists\forall \text{ contains}(/l/\text{vege} \& \text{vegan} \& \text{summer})), \exists \text{ serves}(\exists\forall \text{ contains}(\text{summer})), \exists \text{ serves}(\exists\forall \text{ contains}(\text{vege}))$
 $\Rightarrow \exists \text{ serves}(\exists\forall \text{ contains}(\text{vegan}))$

$\langle 2 \rangle \exists \text{ serves}(\text{IdForest} \& \exists\forall \text{ contains}(\text{autumn})), \exists \text{ serves}(\exists\forall \text{ contains}(\text{summer})), \exists \text{ serves}(\exists\forall \text{ contains}(\text{vege}))$
 $\Rightarrow \exists \text{ serves}(\exists\forall \text{ contains}(\text{vegan}))$

$\langle 4 \rangle \Rightarrow \exists \text{ serves}(\exists\forall \text{ contains}(\text{summer})), \exists \text{ serves}(\exists\forall \text{ contains}(\text{vege}))$

Agenda

Introduction

Formal Concept Analysis & Propositional Logic

Relational Concept Analysis & Description Logics

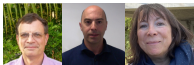
RCA -DL opportunities

Concluding remarks

RCA -DL opportunities

- Ontology building
 - Microbiology (Rokia Bendaoud, Amedeo Napoli, Yannick Toussaint. A Proposal for an Interactive Ontology Design Process based on Formal Concept Analysis. FOIS 2008)
 - Astronomy (Bendaoud, Rokia, Rouane-Hacene, Mohamed, Toussaint, Yannick, Delecroix, Bertrand, Napoli, Amedeo. Construction d'une ontologie à partir d'un corpus de textes avec l'ACF. IC 2007)
- Reasoning (FaCT++, RACER, DLP, Pellet, KAON2)
- Knowledge discovery through populated ontology exploration or implication exploration

Knomana: Plant wisdom



<https://www.knomana.org/en>

- Find alternatives to chemical pesticides and antibiotics, with local plants
- Understanding relations between therapeutic plants, pests (bioagressors), and affected organisms
- Intensive collection of ≥ 48.000 descriptions of plant use in the scientific literature (*Pierre Silvie, Pierre Martin and a community of thematic researchers*)
- Beyond queries: find or hypothesize solutions in phytotherapy by knowledge ewploration (*Pierre Silvie, Pierre Martin, Marianne Huchard and a group of computer scientists*)

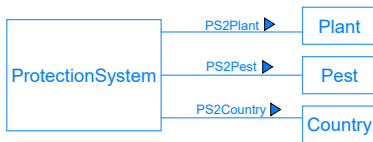


(1) Mature larva of beet armyworm, *Spodoptera exigua* (Hübner). Photograph by John Capinera, University of Florida. (2) Adult beet armyworm, *Spodoptera exigua* (Hübner). Photograph by Lyle J. Buss, University of Florida. (3) *Aspergillus fumigata*
<https://commons.wikimedia.org/wiki/File:Aspergillus.jpg?uselang=fr>

Knomana: Implication exploration



<https://fcavizir.lirmm.fr/>



Excerpt from Knomana

*P1 (Protection System in the field) developed in Namibia (Country):
Cinnamomum zeylanicum (Plant) is used to control Aspergillus flavus (Pest)*

A **protection system** indicates
a **plant** usage to control a **pest** (bioagressor) in a given **country**

Lola Musslin, Alexandre Bazin, Marianne Huchard, Pierre Martin, Pascal Poncelet, Vincent Raveneau, Arnaud Sallaberry: FCAvizIR: Exploring Relational Data Set's Implications Using Metrics and Topics. CONCEPTS 2024: 132-148

Knomana: Implication exploration

A few implications of the Duquenne-Guigues base of implications:

$\exists PS2Pest(aspergillus), \exists PS2Plant(lauraceae \& comestible \& applicOil),$
 $\exists PS2Plant(aromatic \& evergreen)$

$\Rightarrow field, \exists PS2Pest(attacksCheese), \exists PS2Country(australAfrica)$

$\exists PS2Pest(aspergillus), \exists PS2Country(westernAfrica), \exists PS2Plant(aromatic \& evergreen)$

$\Rightarrow lab, \exists PS2Pest(attacksPeanuts),$

$\exists PS2Plant(asteraceae \& toxic \& applicEssentialOil \& applicExtract)$

$\exists PS2Pest(attacksPeanuts), \exists PS2Pest(aspergillus), \exists PS2Plant(aromatic \& evergreen)$

$\Rightarrow lab, \exists PS2Country(westernAfrica),$

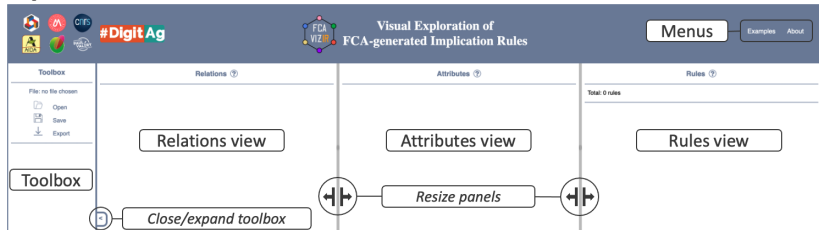
$\exists PS2Plant(asteraceae \& toxic \& applicEssentialOil \& applicExtract)$

(...)

Nota: concepts id are recursively rewritten using their simplified intent (introduced attributes) for the sake of readability by domain experts

Knomana: Implication exploration

<https://fcavizir.lirmm.fr/>



Principles:

- **Filtering** with various **metrics** to rapidly localize implications with certain values of these metrics
- **Clustering** based on **common elements** (relations or attributes) in their premise or conclusion.
- **Estimating** the **quantity** of implications in the formed groups.
- **Navigating** among **topic-based** implication groups by refining/enlarging the selection step-by-step.

Knomana: Implication exploration

<https://fcavizir.lirmm.fr/>



After successive selections:

support ≥ 2 ; relations $\exists PS2Pest \times \{\exists PS2Pest, \exists PS2Plant, \exists PS2Country\}$; attributes $aspergillus \times attacksCheese, attacksCheese \times \{field, australAfrica\}$; premise $\exists PS2Pest(attacksCheese)$

Resulting Implication:

$\exists PS2Pest(attacksCheese), \exists PS2Pest(aspergillus), \exists PS2Plant(aromatic\&evergreen)$
 $\Rightarrow field, \exists PS2Country(australAfrica)$

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Introduction

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RCA -DL opportunities

Concluding remarks

Synthesis

- FCA, RCA: frameworks with dual graphical and logical views
- Transcription in DL frameworks of some RCA settings
- Opportunities for:
 - Ontology building or completion
 - Reasoning
 - Knowledge discovery in a controlled formalism

Future work for efficient use

Conceptual exploration extended to RCA:

- Knowledge elicitation by formal context completion asking implications of a base in a suitable ordering
- FCA framework: Conceptual Exploration, Bernhard Ganter , Sergei Obiedkov, Springer Nature, 2016
- Ask: “Is it true that for all protection systems:
 $\exists PS2Pest(attacksCheese)$, $\exists PS2Pest(aspergillus)$,
 $\exists PS2Plant(aromatic\&evergreen)$
 $\Rightarrow field$, $\exists PS2Country(australAfrica)$ ”
- If **Yes** \rightarrow the system goes to the next implication
- If **No** \rightarrow the expert proposes a new protection system which is added to the relational context family

Future work for efficient use

LLM-assisted RCA output reading (implications, concepts, concept lattices)

E.g. with an Implication

- Explain to the LLM the context, syntax, ... meaning of the relations.
- “Can you write an interpretation in English of the following implication? ”

```
exists_PS2Pest(attacksPeanuts),exists_PS2Pest(aspergillus),  
exists_PS2Plant(aromatic&evergreen) =>  
lab,exists_PS2Country(westernAfrica),  
exists_PS2Plant(asteraceae&toxic&applicEssentialOil&applicExtract)
```

- ChatGPT 4o 2024/01/11:
“Here’s an interpretation of the implication in English, designed for domain experts: (...) If there exists a protection system (PS) that satisfies all of the following conditions: The PS controls a pest that attacks peanuts. The PS controls a pest associated with Aspergillus (a genus of fungi). The PS uses a plant that is both aromatic and evergreen. Then, the following must also hold true: The protection system involves laboratory analysis or has been observed to involve laboratory settings. The protection system has been observed in Western Africa. The protection system uses a plant from the Asteraceae family that is both toxic and is applied in the form of essential oil or extract.(...)”

Future work for efficient use

LLM-assisted Grounding problem:

- “Connect abstract symbols and representations to their real-world meanings and referents.”
- Ask variants of: “what is the term for (SCOPE) which is (ATR1) and (ATR2).... , as (OBJ1), (OBJ2), (OBJ3), , but exclude to (CEX1), nor (CEX2), nor?”
- E.g. / ChatGPT 4o, 2024/01/12:
Question: “what is the term for (C_Pizza_31) which is (Pizza,existForall contains(autumn),existForall contains(vegan))”
Answer: “Here are some suggestions for naming the concept described by (Pizza, $\exists\forall\text{contains(autumn)}\vee\exists\forall\text{contains(vegan)}$) in English:
AutumnVeganPizza: A straightforward and clear name highlighting the key attributes (autumn and vegan). **SeasonalVeganPizza**: Emphasizes the seasonal nature (autumn) and vegan characteristic. **AutumnPlantBasedPizza**: If "plant-based" resonates better than "vegan" in the context. (...)
If you're working within an ontology, AutumnVeganPizza or VeganAutumnPizza would balance clarity and simplicity well. Let me know if you'd like further refinements!”
- G.A. Aranda-Corral and A. Bundy and J. Borrego-Díaz and P.Y. Chan. *Grounding problem in Formal Concept Analysis by means of Large Language Models, Ws. Late Breaking Advances on Conceptual Structures Concepts 2024*

¡Thank you!



<https://rcaviz.lirmm.fr/>
<https://www.lirmm.fr/fca4j/>
<https://fcavizir.lirmm.fr/>

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Grant ANR-16-CONV-0004 **anr***



ANR SmartFCA project
Grant ANR-21-CE23-0023 **anr***