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► **To cite this version:**

Ellen Paixão, Helga Balbi, Esther Pacitti, Fabio Porto, Joel Santos, et al.. FFT-Based Anomaly Detectors: Cutoff Frequency Adjustment and SMA-Based Approach. *Journal of Information and Data Management*, inPress. <lirmm-05332270>

HAL Id: lirmm-05332270

<https://hal-lirmm.ccsd.cnrs.fr/lirmm-05332270v1>

Submitted on 26 Oct 2025

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FFT-BASED ANOMALY DETECTORS: CUTOFF FREQUENCY ADJUSTMENT AND SMA-BASED APPROACH

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ABSTRACT

Abstract This article presents a method for anomaly detection in time series based on the Fast Fourier Transform (FFT) using high-pass filtering. In addition to five existing strategies for determining the cutoff frequency (TF, AF, CAF, BSF, CBSF), a novel approach called SMAF is introduced. SMAF combines spectral analysis with adaptive smoothing using the Simple Moving Average, enabling the detection of high-frequency anomalies without requiring the inverse transform. The experiments employ the Yahoo Webscope dataset and the Numenta Anomaly Benchmark (NAB), providing a comprehensive evaluation. FFT-based approaches are compared to traditional statistical techniques (FBIAD and ARIMA) and machine learning methods (LSTM, ELM, and SVM). The results show that FFT-based methods outperform both statistical and machine learning techniques in terms of F1 score, precision, accuracy, and execution time. Among them, SMAF achieves the highest precision and the lowest execution time, reinforcing the potential of FFT-based filtering for efficient and accurate anomaly detection in time series.

1 Introduction

Anomalies in time series can distort data behavior and introduce bias in parameter estimation, affecting the reliability of subsequent analyses [Erkuş and Purutçuoğlu, 2021]. Identifying and removing such anomalies is essential in domains including finance, industry, geography, and medicine [Yu et al., 2014]. Several detection methods have been proposed based on statistical modeling, machine learning, clustering, and signal decomposition. Statistical techniques such as FBIAD and ARIMA [Lima et al., 2022] detect significant deviations from expected behavior. Machine learning models, including LSTM, ELM, and SVM, aim to capture nonlinear and complex temporal patterns. Clustering algorithms, such as K-means and DBSCAN, identify outliers as elements distant from dense regions. Decomposition-based approaches, including Singular Value Decomposition (SVD) and Principal Component Analysis (PCA), reduce dimensionality and reveal anomalous components [Olteanu et al., 2023].

Another relevant class of methods operates in the frequency domain. Techniques such as the Fast Fourier Transform (FFT) and the Wavelet Transform exploit spectral properties to identify irregularities. Despite their potential, frequency-based methods remain underutilized in time series analysis [Zhou et al., 2023]. FFT converts time-domain data into a frequency-domain representation, supporting the identification of periodicities, trends, and noise filtering. It is widely used in signal processing, telecommunications, and electromagnetic field analysis [Oppenheim et al., 1997]. In anomaly detection, the central assumption is that anomalies alter the frequency content of the series, either by introducing peaks at unexpected frequencies or by modifying the energy distribution [Zhou et al., 2023].

This study extends the work of Silva et al. [2024], which proposes the construction of high-pass filters using FFT to suppress low-frequency components, such as trend and seasonality while preserving high-frequency components

associated with anomalies [Jiang et al., 2021]. The core challenge addressed in that work is the automatic selection of the cutoff frequency without requiring prior knowledge about the temporal structure of the series.

In Silva et al. [2024], five approaches (one baseline and four new ones) are proposed for defining the cutoff frequency. Their performance is evaluated using datasets with varying temporal profiles, taking into account features such as volatility, trend presence, and seasonality. FFT-based methods are compared with statistical techniques (FBIAD and ARIMA) and machine learning models (LSTM, ELM, and SVM), highlighting the need to select detection strategies according to the specific constraints of the application scenario, particularly in terms of accuracy and processing time.

Building on this foundation, the present work introduces a new method that combines FFT-based spectral analysis with adaptive smoothing using a Simple Moving Average (SMA). In this method, the dominant frequency of the signal defines the window size of the moving average, which functions as a low-pass filter. Subtracting the smoothed component from the original signal yields a residual that highlights high-frequency variations, allowing for more precise anomaly detection. This strategy offers adaptability and computational efficiency, representing a practical improvement over previous FFT-based techniques.

The experimental evaluation is also extended to a broader set of datasets, allowing the proposed methods to be validated in diverse operational contexts and improving the robustness of the analysis.

The remainder of this article is structured as follows. Section 2 presents related work. Section 3 describes the FFT-based strategies from Silva et al. [2024] and the new FFT-SMA method. Section 4 details the experimental evaluation. Section 5 presents the conclusions.

2 Literature Review

Collins Jackson and Lacey [2020] demonstrate the use of the Discrete Fourier Transform (DFT) to identify seasonality and anomalies in sparse binary data, proposing a detection method based on the sum of distances. Bürger and Pauli [2013] present an unsupervised approach for anomaly detection and segmentation in sequential data, images, and volumetric datasets, relying on multiscale analysis using only the phase of the Fourier Transform. Herrera et al. [2021] describe a framework for detecting anomalies in internet traffic across core and metropolitan networks by applying time series analysis via the Graph Fourier Transform to improve computational efficiency and precision.

Loyarte and Menenti [2008] analyze the influence of rainfall anomalies on the spectral parameters of the Normalized Difference Vegetation Index (NDVI) time series in northwestern Argentina. Ye et al. [2023] propose the Fourier Time Series Transformer (FTST), a model for anomaly detection in multivariate time series that combines features from temporal and frequency domains to improve detection performance. Lindstrom et al. [2020] introduce functional kernel density estimation (FKDE) techniques for detecting anomalies in aviation-related time series, integrating point-based and Fourier-based methods.

Zhao et al. [2018] develop a method based on the Fourier series to isolate anomalies in power telecommunications network traffic. Bhattacharya et al. [2020] describe an FFT-based approach for detecting and classifying thermoacoustic instability (TAI) and lean blowout (LBO) in turbulent combustors. Erkuş and Purutçuoğlu [2021] propose the Frequency-domain Outlier Detection (FOD) algorithm, which targets quasi-periodic anomalies in time series and shows superior performance compared to traditional methods in both simulations and real-world applications.

Rong and Bailis [2017] introduce Automatic Smoothing for Attention Prioritization (ASAP), a visualization operator that uses an adaptive SMA to smooth time series data while preserving structural deviations. The method reduces local variance and retains major patterns to enhance visual interpretability. It incorporates a roughness metric based on first differences, a structural constraint based on kurtosis, and pre-aggregation techniques guided by screen resolution and autocorrelation for real-time visualization.

Alghamdi et al. [2024] assess the impact of FFT, Butterworth filtering, and SMA on the detection of frequency events in Phasor Measurement Unit (PMU) data. Their study evaluates the effect of these denoising techniques on the performance of a statistical detection algorithm, which is based on mean, variance, and standard deviation rather than traditional metrics such as SNR or RMSE. Results indicate that FFT and SMA are viable alternatives to the Discrete Wavelet Transform (DWT), with SMA showing effectiveness in attenuating rapid fluctuations while preserving relevant signal characteristics.

These studies underscore the importance of FFT-based techniques in anomaly detection tasks across various domains. However, the selection of the cutoff frequency remains an underexplored aspect, pointing to the need for further investigation, as emphasized by Silva et al. [2024]. Although prior work has compared FFT- and SMA-based approaches, the potential advantages of combining both techniques to enhance data quality and pattern detection have not been thoroughly examined. This article contributes to advancing this research direction.

3 FFT-Based Anomaly Detectors

Let X be a time series containing n observations, such that $X = \langle x_1, \dots, x_n \rangle$. Let Y be the frequency-domain representation of the time series obtained from an FFT, such that $Y = \text{FFT}(X)$. Consider h to be a high-pass filter that adjusts its cutoff frequency based on the power spectrum P of Y , computed as $P = |Y|^2$. After calculating P , a cutoff frequency θ_c is determined for the filter. When applying h to Y , it yields a filtered frequency-domain signal \dot{Y} in which frequency components below the threshold θ_c are removed. The core challenge addressed in this work is the investigation of alternative strategies for selecting the cutoff frequency f .

Once the cutoff frequency is defined, one can apply the Inverse FFT (IFFT) to obtain the filtered time series in the time domain, denoted as $\omega = \langle \omega_1, \dots, \omega_n \rangle$. This residual series ω is expected to exclude low-frequency components associated with trend and seasonality in X . Outliers in ω are identified as observations ω_t that deviate significantly from the interquartile range, as defined by Equation 1, where $IQR(\omega)$ is the interquartile range of ω , and $Q_1(\omega)$ and $Q_3(\omega)$ are the first and third quartiles, respectively. These outliers can be mapped back to the original time series as anomalies since they occur at the same time indices, *i.e.*, $anomalies(X) = outliers(\omega)$ [Ogasawara et al., 2025b].

$$outliers(\omega) = \{t \mid \omega_t \notin [Q_1(\omega) - 1.5 \cdot IQR(\omega), Q_3(\omega) + 1.5 \cdot IQR(\omega)]\} \quad (1)$$

In addition to frequency-domain filtering, this work proposes an alternative approach that applies high-pass filtering through adaptive smoothing using SMA. In this method, the periodogram of X is used to identify the dominant frequency θ_{dom} in the signal. Based on this frequency, an optimal window size k is computed as the inverse of θ_{dom} , *i.e.*, $k = \lfloor 1/\theta_{dom} \rfloor$, and is used to smooth the original series using an SMA. The smoothed signal \hat{X} captures the low-frequency components, and the residual signal is calculated as $\omega = X - \hat{X}$, which emphasizes the high-frequency variations. As in the previous method, anomalies are detected by identifying outliers in ω according to Equation 1 and mapped back to the corresponding time points in X .

The threshold defined in Equation 1 is derived from the classical boxplot rule introduced by Tukey [1977], in which $1.5 \times IQR$ is used to identify mild outliers.

3.1 High-Pass Filtering in the Frequency Domain

Algorithm 1 summarizes the anomaly detection process using FFT. It begins by taking a time series X as input. First, FFT is applied to X , generating Y . Next, the high-pass filter h is applied to Y , resulting in \dot{Y} . Then, the inverse FFT is applied to \dot{Y} , producing ω . Finally, outliers identified in ω are characterized as anomalies (A). In this procedure, the parameter h represents the high-pass filter constructed using one of the following strategies for defining the cutoff frequency: TF, AF, BSF, CAF, or CBSF. These strategies are described throughout this section.

Algorithm 1 Anomaly Detection Using FFT

- 1: **procedure** ANOMALYFFT(X, h)
 - 2: $Y \leftarrow \text{FFT}(X)$
 - 3: $\dot{Y} \leftarrow h(Y)$ ▷ High-pass filter in frequency domain
 - 4: $\omega \leftarrow \text{IFFT}(\dot{Y})$
 - 5: $A \leftarrow outliers(\omega)$
 - 6: **return** A
-

This work presents five approaches (one baseline and four new ones) for adjusting the cutoff frequency of the high-pass FFT filter h : traditional (TF), AMOC (AF), BinSeg (BSF), CUSUM-AMOC (CAF), and CUSUM-BinSeg (CBSF). Except for the baseline TF, all are based on the assumption that cutoff frequencies can be identified as change points in the power spectrum. The methods are detailed below.

In the TF approach, the cutoff frequency θ_c is initialized with the index of the maximum value of P , *i.e.*, the frequency component that contributes the greatest power to Y . Assuming variation in the values of P , a threshold is defined as the mean plus 2.698 times the standard deviation of P , following the Central Limit Theorem. Values of P below this threshold are adjusted to facilitate the identification of a significant cutoff point. The cutoff frequency θ_c then corresponds to the smallest index in the adjusted spectrum above this threshold.

In the AF approach, the AMOC (At Most One Change) method is used to identify a single significant change in the mean of P . As described in [Killick and Eckley, 2014], this method iteratively partitions P into two segments at each

index, computes the mean of each segment and selects the index that maximizes the difference in means according to a statistical test. The significance of the change is evaluated to ensure it is not due to random fluctuations [Lykou et al., 2020]. The cutoff frequency θ_c is defined as the change point detected in P .

The BSF approach applies the Binary Segmentation (BinSeg) method to detect multiple change points in the mean of P . BinSeg recursively splits the series and searches for points that minimize the within-segment squared errors. For each candidate split, it calculates the mean and total deviation within segments and identifies points where there is a significant change in mean. The process continues until a stopping criterion is met. In this approach, the cutoff frequency θ_c corresponds to the last change point detected in P .

The CAF approach combines the Cumulative Sum (CUSUM) method with AMOC. CUSUM accumulates deviations from a reference mean to detect small but consistent changes. Applying CUSUM to P produces a transformed series \hat{P} , in which regions of structural change are highlighted. AMOC is then applied to \hat{P} to determine the cutoff point. The final θ_c is the change point found in \hat{P} .

The CBSF approach is similar to CAF but uses the BinSeg method instead of AMOC after applying CUSUM. BinSeg analyzes the CUSUM-transformed power spectrum \hat{P} , and the last change point found is assigned as the cutoff frequency θ_c .

3.2 High-Pass Filtering Based on SMA Smoothing

In addition to frequency-domain filtering through direct manipulation of the Fourier spectrum, this work introduces an alternative anomaly detection approach based on adaptive smoothing using an SMA. The goal of this method, named SMAF, is to isolate the high-frequency components of the time series X by subtracting a smoothed version of it, thereby implicitly applying a high-pass filter.

The method begins by computing the periodogram P of the input time series X , which estimates the spectral density of the signal. From P , the dominant frequency θ_{dom} is identified. This frequency corresponds to the point of highest spectral power in the periodogram. It reflects the main periodic component of the time series.

Next, the window size k for the SMA is adaptively defined based on θ_{dom} . The assumption is that a window size inversely proportional to θ_{dom} is effective in isolating the lower-frequency components. Specifically, $k = \lfloor 1/\theta_{dom} \rfloor$. This dynamic adjustment enables the method to adapt to the varying characteristics of the input series without requiring prior knowledge of seasonality or trends.

Once k is defined, the SMA is applied to X , generating a smoothed signal \hat{X} that captures the low-frequency behavior of the series. By subtracting \hat{X} from the original signal, i.e., $\omega = X - \hat{X}$, a residual signal is obtained that emphasizes high-frequency variations and potential anomalies.

Algorithm 2 summarizes this approach. Unlike the direct frequency-domain filtering presented in Section 3.1, this method avoids applying the inverse FFT. Instead, it relies on time-domain smoothing guided by spectral information. The novelty lies in the automatic determination of the SMA window size based on the dominant frequency of the signal, enabling adaptive and efficient high-pass filtering for anomaly detection.

Algorithm 2 Anomaly Detection Using FFT and SMA

```

1: procedure ANOMALYFFT_SMA( $X$ )
2:    $P \leftarrow$  Periodogram( $X$ )
3:    $\theta_{dom} \leftarrow$  DominantFrequency( $P$ )
4:    $k \leftarrow$  WindowSize( $\theta_{dom}$ )
5:    $\hat{X} \leftarrow$  SMA( $X, k$ )
6:    $\omega \leftarrow X - \hat{X}$  ▷ High-pass filtering by subtraction
7:    $A \leftarrow$  outliers( $\omega$ )
8:   return  $A$ 

```

4 Results

This section presents the evaluation of the proposed anomaly detection methods introduced in Section 3. For comparative purposes, other established approaches based on different anomaly detection paradigms are also considered. These include statistical methods (FBIAD and ARIMA) and machine learning models (LSTM, ELM, and SVM). All methods are implemented in the *Harbinger* R package, available on CRAN Ogasawara et al. [2025a]. The sliding

window size is set to 30 for FBIAD. For LSTM, the number of *epochs* is set to 10,000. The *actfun* parameter is configured as *Purelin* for ELM, and the SVM model uses the *Radial Basis* kernel.

Two datasets are employed: the Yahoo Labs dataset and the Numenta Anomaly Benchmark (NAB).

The Yahoo Labs dataset comprises a set of time series, including both synthetic and real data associated with traffic patterns from Yahoo’s services¹. These series exhibit diverse behaviors, providing a representative benchmark for comparison. Anomalies are labeled, allowing for a precise evaluation of the methods.

The NAB dataset² also includes a wide set of labeled time series designed for testing anomaly detection algorithms. It is widely recognized as a standard benchmark in the field. The dataset includes both real-world signals, such as AWS infrastructure metrics, and synthetic series. Its inclusion enables the assessment of methods in real-time and cloud computing scenarios. All experiments are conducted on an Intel Xeon w3-2423 processor with 512 GB of RAM, 12 cores, and Ubuntu 22.04 LTS.

The evaluation considers precision, recall, F1 score, accuracy, and execution time, all of which are derived from the confusion matrix. Precision indicates the proportion of true positives among predicted positives. Recall reflects the proportion of actual anomalies correctly identified. The F1 score is the harmonic mean of precision and recall. Accuracy refers to the proportion of correct classifications. Execution time quantifies computational efficiency. All reported metrics represent the average values across the evaluated time series.

Table 1 shows the results for the Yahoo dataset, and Table 2 presents the results for NAB. Bold values denote the best result for each metric, and underlined values indicate the second-best.

Table 1: Results for Yahoo Dataset

Method	Precision	Recall	F1	Accuracy	Time (s)
FBIAD	0.14	0.69	0.23	0.94	9.01
ARIMA	0.06	<u>0.67</u>	0.11	0.93	130.94
LSTM	0.07	<u>0.64</u>	0.13	0.93	1280.58
ELM	0.06	0.64	0.11	0.93	4.55
SVM	0.04	0.66	0.08	0.91	73.80
TF	0.49	0.33	0.39	<u>0.98</u>	1.81
AF	0.41	0.35	0.38	<u>0.98</u>	2.09
BSF	0.22	0.44	0.29	<u>0.98</u>	2.86
CAF	<u>0.54</u>	0.42	0.47	<u>0.98</u>	2.01
CBSF	0.24	0.48	0.32	0.97	2.26
SMAF	0.78	0.32	<u>0.45</u>	0.99	<u>1.92</u>

Table 2: Results for Numenta Dataset

Method	Precision	Recall	F1	Accuracy	Time (s)
FBIAD	0.01	0.87	0.02	0.93	4.64
ARIMA	0.01	0.85	0.02	0.93	66.06
LSTM	0.01	0.76	0.01	0.92	379.50
ELM	0.01	0.78	0.02	0.92	1.54
SVM	0.01	0.77	0.02	0.92	218.28
TF	0.02	<u>0.86</u>	<u>0.03</u>	0.96	0.80
AF	0.02	0.84	0.04	0.96	<u>0.79</u>
BSF	0.01	0.87	0.02	0.94	0.88
CAF	0.01	0.76	0.02	0.94	0.81
CBSF	0.01	0.70	0.02	<u>0.95</u>	0.88
SMAF	0.01	0.73	<u>0.03</u>	0.96	0.46

For both datasets, the FBIAD method achieves a high recall, indicating its ability to detect anomalies effectively. It reaches 0.69 on Yahoo and 0.87 on NAB. However, its low precision (0.14 and 0.01, respectively) reveals a high false-positive rate, resulting in low F1 scores despite high accuracy and an acceptable processing time.

¹<https://yahoresearch.tumblr.com/post/114590420346/>

²<https://github.com/numenta/NAB>

ARIMA obtains similar recall values (0.67 on Yahoo and 0.85 on NAB) but also suffers from low precision and F1 score. Its execution time is among the highest, particularly on Yahoo (130.94s), which may limit its suitability for real-time tasks.

The LSTM model shows limited performance, with very low F1 scores (0.13 on Yahoo and 0.01 on NAB) and the highest execution times (1280.58s on Yahoo and 379.50s on NAB). These results suggest inefficiency and poor detection capability under the evaluated conditions.

Among machine learning models, ELM and SVM perform moderately. ELM attains reasonable recall (0.64 on Yahoo, 0.78 on NAB) but low precision. SVM exhibits similar behavior, albeit with a higher computational cost (218.28s on NAB), rendering it less viable for deployment.

The proposed FFT-based methods (TF, AF, CAF, BSF, CBSF, SMAF) exhibit significantly better overall performance. On Yahoo, TF and AF yield high precision (0.49 and 0.41) and accuracy (0.98), with low execution times (1.81 s and 2.09 s). On NAB, they maintain good recall (0.86 for TF and 0.84 for AF) and low processing time, with F1 scores superior to those of baseline methods. AF achieves the highest F1 score (0.04) on NAB.

CAF presents balanced results. On Yahoo, it yields the best F1 score (0.47), precision (0.54), and accuracy (0.98), with low execution time. On NAB, it achieves high accuracy (0.94), moderate recall (0.76), and low latency (0.81s), reinforcing its robustness.

SMAF stands out with the highest precision (0.78) and accuracy (0.99) on Yahoo and the lowest execution time (0.46s) on NAB. While its F1 scores are not the highest (0.45 and 0.03), they remain competitive. SMAF's lower recall indicates a conservative detection profile, as it emphasizes sharper variations. In contrast, ELM is more sensitive but yields a higher number of false positives. Future work may explore hybrid methods to combine SMAF's precision with broader recall coverage.

In summary, deep learning models, such as LSTM, exhibit limitations due to their high computational cost and low effectiveness. Traditional methods, such as ARIMA and SVM, also underperform in terms of precision and recall. The FFT-based methods, particularly CAF and SMAF, combine high accuracy, efficiency, and interpretability, making them well-suited for anomaly detection in time series.

5 Conclusion

This article presents an extended study on high-pass filtering strategies based on the FFT for anomaly detection in time series. In addition to five previously proposed strategies for cutoff frequency adjustment (TF, AF, BSF, CAF, and CBSF), this work introduces SMAF, a new method that combines spectral analysis with adaptive smoothing using an SMA. Unlike traditional FFT-based approaches, SMAF does not require the inverse transform, which improves computational efficiency while preserving sensitivity to high-frequency variations commonly associated with anomalies.

The evaluation is conducted using two well-established datasets: Yahoo Webscope and the NAB. The Yahoo dataset includes seasonal and structured series, while NAB contains shorter and irregular anomalies with real-time constraints.

On the Yahoo dataset, SMAF achieves the highest precision (0.78) and accuracy (0.99), along with one of the lowest execution times. These results demonstrate the method's effectiveness in reducing false positives while maintaining detection reliability. On the NAB dataset, SMAF achieves the best execution time (0.46s) and a competitive F1 score. Although methods such as AF and CAF obtain higher F1 scores in this dataset, SMAF provides the best trade-off between accuracy and computational cost, making it particularly suitable for scenarios that require fast and precise anomaly detection.

The results confirm that FFT-based methods consistently outperform both statistical techniques (FBIAD and ARIMA) and machine learning models (LSTM, ELM, and SVM), particularly in terms of precision and efficiency. Among them, CAF and AF maintain balanced performance across datasets. At the same time, SMAF stands out for its simplicity, adaptability, and execution speed.

These findings reinforce the relevance of FFT-based anomaly detection strategies, especially those incorporating adaptive mechanisms such as SMAF. In future work, we intend to explore hybrid and ensemble approaches that combine the strengths of precision-oriented and recall-oriented detectors and to extend the evaluation to multivariate and highly non-stationary time series.

Acknowledgments

The authors thank CAPES, CNPq, and FAPERJ for partially supporting this paper.

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